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Derivation and application of the fully magnetized kinetic equations*

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In the magnetized and laser fusion plasma, space and astrophysical plasma, the particles' gyro-radii can be smaller than the Debye length when there is a strong magnetic field. This will have a significant influence on collision dynamics and many physical processes such as parallel velocity slowing down, temperature relaxation, particle diffusion, thermal transport, and so on..

The fully magnetized Fokker-Planck equation is derived by including a uniform magnetic field in the collision term as follows:

$$\frac{\partial f_\alpha(\mathbf{v}_\alpha,\tau)}{\partial\tau}+\Omega_\alpha\mathbf{v}_\alpha\times\hat{\mathbf{e}}_z\cdot\frac{\partial f_\alpha(\mathbf{v}_\alpha,\tau)}{\partial\mathbf{v}_\alpha}$$

$$=-\frac{\partial}{\partial\mathbf{v}_\alpha}\cdot[\langle\Delta\mathbf{V}_\alpha\rangle f_\alpha(\mathbf{v}_\alpha,\tau)]$$

$$+\frac{1}{2}\frac{\partial^2}{\partial\mathbf{v}_\alpha\partial\mathbf{v}_\alpha}:[\langle\Delta\mathbf{V}_\alpha\Delta\mathbf{V}_\alpha\rangle f_\alpha(\mathbf{v}_\alpha,\tau)]$$

Where the magnetized Fokker-Planck coefficients $\langle\Delta\mathbf{V}_\alpha\rangle$ and $\langle\Delta\mathbf{V}_\alpha\Delta\mathbf{V}_\alpha\rangle$ have been derived explicitly within the binary collision model and the fully magnetized Landau equation is obtained:

$$\frac{\partial f_\alpha(\mathbf{v}_\alpha,\tau)}{\partial\tau}+\Omega_\alpha\mathbf{v}_\alpha\times\hat{\mathbf{e}}_z\cdot\frac{\partial f_\alpha(\mathbf{v}_\alpha,\tau)}{\partial\mathbf{v}_\alpha}$$

$$=\frac{\partial}{\partial\mathbf{v}_\alpha}\cdot\sum_\beta(2\pi)^3\frac{q_\alpha^2q_\beta^2}{m_\alpha}\int_0^\infty dt_2\int\mathrm{d}^3\mathbf{k}\int\mathrm{d}^3\mathbf{v}_\beta$$

$$\times\tilde{\Phi}_D^2(k)\exp\{\mathrm{i}\mathbf{k}\cdot[\mathbf{H}_\alpha(t_2)-\mathbf{H}_\alpha(0)]\cdot\mathbf{v}_\alpha$$

$$-\mathrm{i}\mathbf{k}\cdot\left[\mathbf{H}_\beta(t_2)-\mathbf{H}_\beta(0)\right]\cdot\mathbf{v}_\beta\}\mathbf{T}_\alpha^{-1}(t)\cdot\mathbf{kk}$$

$$\times\left(\frac{1}{m_\alpha}\frac{\partial}{\partial\mathbf{v}_\alpha}-\frac{1}{m_\beta}\frac{\partial}{\partial\mathbf{v}_\beta}\right)\left[f_\alpha(\mathbf{v}_\alpha,\tau)f_\beta(\mathbf{v}_\beta,\tau)\right]$$

where $\tilde{\Phi}_D(k)$ is the Fourier transform of Debye screened potential and,

$$\mathbf{H}_\alpha(t)=\frac{1}{\Omega_\alpha}\begin{pmatrix}\sin(\Omega_\alpha t) & -\cos(\Omega_\alpha t) & 0\\ \cos(\Omega_\alpha t) & \sin(\Omega_\alpha t) & 0\\ 0 & 0 & \Omega_\alpha t\end{pmatrix}$$

The above kinetic equation is shown to be identical to the result obtained from the BBGKY approach when the collective effects are neglected and satisfy the conservation of particles, momentum, and energy. [1]

The fully magnetized Balescu-Lenard-Guernsey equation is derived by including a uniform magnetic field in collision term and by employing the Fokker-Planck approach. By using the fluctuating electrostatic field for quiescent plasmas, the magnetized Fokker-Planck coefficients are calculated explicitly based on the wave theory which includes the collective effects in a proper manner. The coefficients of the polarization and the fluctuations correlation are respectively as

Manipulating the magnetized Fokker-Planck collision term into the Landau form, the magnetized Balescu-Lenard-Guernsey collision term is obtained as follows [2]:

$$\frac{\partial f_\alpha(\mathbf{v}_\alpha,\tau)}{\partial\tau}$$

$$=\frac{\partial}{\partial\mathbf{v}_\alpha}\cdot\sum_\beta\frac{q_\alpha^2q_\beta^2}{m_\alpha}\int_{-\infty}^\infty\mathrm{d}t\int_{-\infty}^\infty dt_1\int\mathrm{d}^3\mathbf{k}\int_{-\infty}^\infty\mathrm{d}\omega\int\mathrm{d}^3\mathbf{v}_\beta$$

$$\times\exp\{\mathrm{i}\mathbf{k}\cdot[\mathbf{H}_\alpha(t)-\mathbf{H}_\alpha(0)]\cdot\mathbf{v}_\alpha$$

$$-\mathrm{i}\mathbf{k}\cdot\left[\mathbf{H}_\beta(t_1)-\mathbf{H}_\beta(0)\right]\cdot\mathbf{v}_\beta-\mathrm{i}\omega(t-t_1)\}$$

$$\times\frac{\mathbf{k}}{2(2\pi)^4\varepsilon_0^2|\varepsilon(\mathbf{k},\omega)|^2k^4}\left[\mathbf{k}\cdot\mathbf{T}_\alpha(t)\frac{1}{m_\alpha}\frac{\partial}{\partial\mathbf{v}_\alpha}\right.$$

$$\left.-\mathbf{k}\cdot\mathbf{T}_\beta(t)\frac{1}{m_\beta}\frac{\partial}{\partial\mathbf{v}_\beta}\right]\left[f_\alpha(\mathbf{v}_\alpha,\tau)f_\beta(\mathbf{v}_\beta,\tau)\right]$$

It is shown that the impact of strong magnetic field is significant on transport processes such as stopping power and temperature relaxation etc.

References

[1] Chao Dong, Wenlu Zhang, and Ding Li, Phys. Plasmas 23 (8), 082105, 2016.
[2] Chao Dong, Wenlu Zhang, Jintao Cao, and Ding Li, Phys. Plasmas 24 (12), 122120, 2017.

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