

Thermal convection and induced mean zonal flows in rotating spherical shells

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Here, we discuss problems of thermal convection in rotating spherical shells and its mean zonal flow generation, which have been studied as fundamental model of internal and surface flows of celestial bodies since 1960's.

Patterns of critical thermal convection in rotating spherical shells propagate retrograde in cases of the slow rotation rate of the shell, while they propagate prograde in rapidly rotating cases. These features can be interpreted as a propagation mechanism of Rossby waves, which is caused by stretching and shrinking of vortex tubes in a rotating frame of reference. The structure of critical convection extends along the spherical shell when the rotation rate is small (banana-type structure). In contrast, when the rotation rate is large, the structure of critical convection elongates in the direction of the rotating axis (columnar structure). This difference of convective structure causes opposite relations between inward/outward flows and stretching/shrinking of vortex tubes, resulting inverse propagation directions (Takehiro 2010). Spiralling and tilting structures of convection cells emerging in rapidly rotating shells can be understood through characteristics of structure and propagation of Rossby waves (Takehiro 2008).

When the rotation rate of the spherical shells is small, the direction of mean zonal flows induced by critical convection at the equatorial outer boundary changes from prograde to retrograde as the value of the Prandtl number and/or the thickness of convective layer increases. Dependency of the induced mean zonal flow on the Prandtl number can be understood by comparing the relative strength between the transports of angular momentum due to mean meridional circulation and due to Reynolds stress. When the Prandtl number is small, the angular momentum transport due to Reynolds stress determines the direction of mean zonal flows, while, when the Prandtl number is large, the angular momentum transport by the induced mean meridional circulation becomes a dominant factor (Takehiro and Hayashi 1999).

Dependency of the direction of induced mean zonal flow on the layer thickness is caused by the existence of a state where the acceleration by the angular momentum transport due to Reynolds stress actually induces a flow

in the opposite direction to that of the enforced acceleration. Because of the Coriolis force, the zonal acceleration enhances meridional circulation, which is enhanced by the thermal effect and, through the further effect of the Coriolis force, causes zonal wind in the opposite direction (Takehiro and Hayashi 1999).

When the rotation rate is large, the direction of the mean zonal flows at the equatorial outer boundary does not depend on the layer thickness, although it still shows the dependency on the Prandtl number which is similar to that of when the rotation rate is small. Since, when the rotation rate is large, the critical modes of convection have an elongated structure in the direction of the rotation axis, the distribution of the prograde/retrograde acceleration by Reynolds stress is also uniform along the rotation axis. As a result of this, mean meridional circulations are not induced, and the zonal wind in the opposite direction to the acceleration does not emerge. (Takehiro and Hayashi 1999).

When the rotation rate is large, characteristics of mean zonal flows induced by finite amplitude convection is similar to those by critical convection. When the Prandtl number is $O(1)$ or less, equatorial prograde flows at the outer boundary emerge due to Reynolds stresses, while equatorial retrograde flows are induced by angular momentum transport of mean meridional flows in the cases with large Prandtl numbers. However, when the amplitude of convection is further increased, equatorial flows at the outer boundary become retrograde even when the Prandtl number is small due to angular momentum homogenization.

References

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