

Fully conservative scheme for Braams and Karney potential equation

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For decades, the numerical violation of conservation laws has been one of the big issues on kinetic plasma simulations. Recently, we have developed a relativistic Vlasov–Maxwell scheme that can strictly maintain the conservation laws of charge, momentum and energy at the round-off level [1]. When constructing the algorithm, the mathematical formulae of product rule, integration-by-parts and commutative rule of partial differential operators should be preserved, even in the discrete level.

The conservation property is also important when the binary collision is included in kinetic simulations. In particle-in-cell simulations, the conservation laws of momentum and energy cannot be maintained strictly if the collided particles have different weights. In the rejection method of Nanbu and Yonemura, both momentum and energy are not conserved owing to the statistic errors [2]. In the margining method of Sentoku and Kemp, the momentum conservation is violated although the law of energy conservation is strictly maintained [3]. On the other hand, some conservative schemes have been developed for the Vlasov simulation. In the gyrokinetic simulations, a conservative collision operator was constructed by Satake et al. while the Fokker–Planck collision terms are linearized because of the huge computational costs [4]. Recently, Taitano et al. developed a fully conservative scalable scheme for the Landau–Fokker–Planck equation without any linearization [5]. They discretized the Rosenbluth potential [6] rather than the Landau–Fokker–Planck equation so that the computational costs required to calculate the collision kernel is reduced dramatically. The Rosenbluth potential is strongly depends on the delta function which is hardly to be dealt with in numerical simulations. They introduced some kind of error factors to the fluxes in momentum space that can recover the conservation property.

In this work, the fully conservative scalable algorithm is extended to the relativistic Landau–Fokker–Planck equation. A potential form of the relativistic Landau–Fokker–Planck equation was proposed by Braams and Karney [7]. If the numerical solution of the relativistic potential is obtained, one can construct a fully conservative scheme with the similar way of Ref. [5]. However, the potential equation is described as an elliptic partial differential equation in general form:

$$A \frac{\partial^2 \Psi}{\partial x^2} + 2B \frac{\partial^2 \Psi}{\partial x \partial y} + C \frac{\partial^2 \Psi}{\partial y^2} + D \frac{\partial \Psi}{\partial x} + E \frac{\partial \Psi}{\partial y} + F \Psi = (\text{RHS}).$$

In a numerical simulation including mixed derivatives, the numerical solution diverges without a scheme which can maintain monotonicity [8]. Moreover, the linear equation corresponding to the elliptic equation has a non-diagonal dominant matrix. Usually, a multigrid method uses, for example, the Gauss–Seidel method as a smoother, which is applicable only to diagonal dominant matrices. We implemented one of the Arnoldi method as a smoother, so the elliptic equation in general form can be solved by a scalable algorithm (Fig. 1). We will construct a fully conservative scalable scheme for relativistic Landau–Fokker–Planck equation by using these tools.

References

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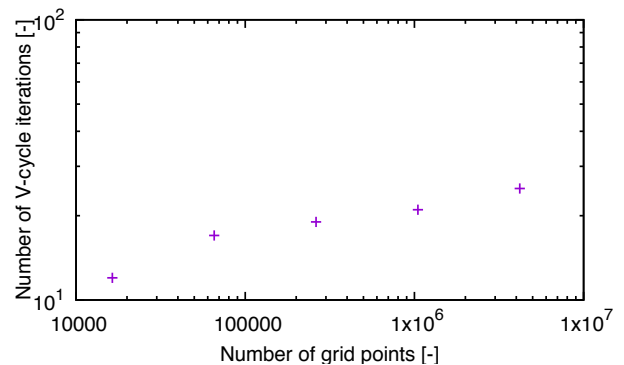


Figure 1. Weak scaling of multigrid method for stiff non-diagonal dominant matrices.