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Classical von Neumann Entropy - a Measure of Phase Randomization of Wave Fields in Turbulence -

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We propose a new measure of phase-randomizing degree of wave fields in turbulent plasma states. The idea is application of the concept of the von Neumann entropy to classical wave turbulence systems and it is a natural extension of the Gibbs entropy. For that purpose, a concept of a density matrix of classical wave fields is introduced together. We show validity of the classical von Neumann entropy to distinguish turbulent states and coherent state having broad spectrum quantitively [1].

In the field of wave turbulence, wave action, which represents the number of waves as a function of a quantum number such as frequency or wave number, has been a subject of investigation. For derivation of wave kinetic equation, which determines time evolution of the spectral distribution of wave action density, conventionally random-phase approximation (RPA) is employed for the closure of the hierarchy of the governing equations [2, 3]. From the kinetic equation, the H-theorem can be derived by defining the wave entropy $S_{wave} \equiv \int \ln(n_k) d\boldsymbol{k}$, where n_k and $\boldsymbol{k}$ are the wave action density and the wave vector, respectively [4]. Since the definition of the wave entropy is obviously based on RPA, S_{wave} is inapplicable to wave phenomena which retain phase coherence. Moreover, physical interpretation of S_{wave} is unclear. Therefore we propose a new idea representing wave field entropy, that doesn't require RPA. Examples, to which our idea is fit, include supercontinuum, optical turbulence, rogue waves, drift wave turbulence and so on, that are recognized as wave turbulence.

We examine an application of the von Neumann entropy to time series data sets to see its validity. Two data sets are prepared: One is white noise and the other is nonlinearly interacting three sinusoidal waves ψ_{3wave} expressed as

$$\psi_{3-wave} = \cos(\omega_1 t + \phi_1) + \cos(\omega_2 t + \phi_2) + \cos(\omega_1 t + \phi_1)\cos(\omega_2 t + \phi_2) + rand.$$

Here, ω_i and ϕ_i are the angular frequency and the initial phase of the i-th mode, respectively. The third term in the right hand side (rhs) represents the nonlinear coupling term between the 1st and 2nd modes. In the rhs small noise expressed as *rand* is added.

Figure 1 shows (a), (b) the power spectra, (c), (d) bicoherence $|b|^2$and (e), (f) the amplitude of density matrices $|\rho|$ of the white noise and $\psi_{3\text{-wave}}$, respectively. The bicoherence, $|b|^2$, is a measure of the fraction of the total product of powers of the frequency trios (ω_1, ω_2, $\omega_1 \pm \omega_2$) that is caused by phase-coupled three modes, and it is given by $|b|^2(\omega_1,\omega_2) = |B(\omega_1,\omega_2)|^2$, where B is the bispectrum defined as $|B(\omega_1,\omega_2)| = \frac{E[X(\omega_1)X(\omega_2)X^*(\omega_1+\omega_2)]}{\sqrt{P_{1,2}(\omega_1,\omega_2)P(\omega_1+\omega_2)}}$, and E is the expectation operator that averages over an ensemble of realizations, X is the Fourier transform of a realization of the time series data, and P is the power defined as $P_{1,2}(\omega_1,\omega_2) = E[X(\omega_1)X(\omega_2)X^*(\omega_1)X^*(\omega_2)]$.

As the white noise has no correlation between any modes as shown by the $|b|^2$ [Fig. 1 (c)], the amplitude of the density matrix $|\rho|$ clearly shows diagonal components only, indicating that the state is highly mixed state. On the other hands, for $\psi_{3\text{-wave}}$, the off-diagonal components of $|\rho|$ are not smeared out [Fig. 1 (f)]. The respective $S_{Neumann}$ are 3.70 for the white noise and 0.09 for $\psi_{3\text{-wave}}$. The maximum $S_{Neumann}$ in this analysis is $\ln(N_{mode})$ = ln(1000) ≒ 6.91.

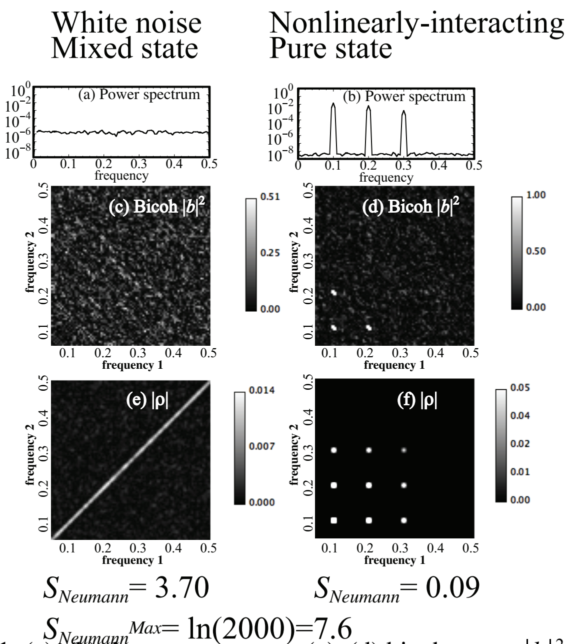

Fig.1. (a), (b) the power spectra, (c), (d) bicoherence $|b|^2$ and (e), (f) the amplitude of density matrices $|\rho|$ of the white noise and $\psi_{3\text{-wave}}$, respectively.

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