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Kinetic understanding of self-organization in long-range interacting systems evidenced by numerical simulations on PEZY-SC

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Our final goal is to reveal a mechanism of self-organization in long-range interacting systems. One of our interests goes to a large-scale vortex formation. In this context, nonneutral plasma (NNP) experiments have played an important role as a two-dimensional motion of electrons confined axially by end electrostatic potentials and radially by a strong magnetic field is described by a two-dimensional, inviscid Euler equation. NNP experiments have provided various phenomena including a vortex crystal formation [1-3], vortex merger [4,5] , vortex hole formation [6,7]. To understand these phenomena, we have carried out numerical simulations with two-dimensional point vortex model which is convenient mathematically and numerically. Despite its simplicity, the model has all the feature required to investigate a long-range interacting n-body systems.

We will mainly present two topics in a 2D point vortex system: (i) Fokker-Planck-type collision term consisting from a diffusion term and a drift term (ii) two-body correlation function that describes a hole around a strong vortex observed in nonneutral plasma experiment.

(i) Fokker-Planck-type collision term

Let us consider a system with N_+ positive and N_- negative point vortices. Circulation of each vortex is either Ω or - Ω where Ω is a positive constant. The position vector of the *i*-th vortex is given by $\boldsymbol{r}_i$.

$$\hat{\omega}(\boldsymbol{r},t) = \hat{\omega}_+(\boldsymbol{r},t) + \hat{\omega}_-(\boldsymbol{r},t),$$

$$\hat{\omega}_+(r,t) = \sum_{i=1}^{N_+} \Omega\delta(\boldsymbol{r}-\boldsymbol{r}_i), \quad \hat{\omega}_-(r,t) = -\sum_{i=N_++1}^{N_++N_-} \Omega\delta(\boldsymbol{r}-\boldsymbol{r}_i)$$

The hat sign represents the variable is a microscopic one. The point vortices are formal solution of the 2D Euler equation.

$$\frac{\partial\hat{\omega}_\pm}{\partial t} + \nabla\cdot(\hat{\boldsymbol{u}}\hat{\omega}_\pm) = 0, \quad \hat{\boldsymbol{u}} = -\hat{\boldsymbol{z}}\times\nabla\hat{\psi}$$

$$\hat{\psi} = \sum_i \Omega_i G(\boldsymbol{r}-\boldsymbol{r}_i), \quad \mathrm{G}(\boldsymbol{r}) = \frac{1}{2\pi}\ln|\boldsymbol{r}|^{-1}$$

Note that two independent equations are abbreviated to the above double-sign single equation for simplicity. Applying the Klimontovich formalism [8] and expressing the equations in the form of the perturbation expantion, we obtain explicit formulae [9]

$$\frac{\partial\omega_\pm}{\partial t} + \nabla\cdot(\boldsymbol{u}\omega_\pm) = -\nabla\cdot(-\mathbf{D}_s\cdot\nabla\omega_\pm \pm \boldsymbol{V}_s\omega_\pm)$$

$$\mathbf{D}_s = K\int d\boldsymbol{r}'\frac{(\boldsymbol{u}-\boldsymbol{u}')(\boldsymbol{u}-\boldsymbol{u}')(\omega'_+ - \omega'_-)}{|\boldsymbol{u}-\boldsymbol{u}'|^3}$$

$$\boldsymbol{V}_s = K\int d\boldsymbol{r}'\frac{(\boldsymbol{u}-\boldsymbol{u}')(\boldsymbol{u}-\boldsymbol{u}')\cdot\nabla\omega')}{|\boldsymbol{u}-\boldsymbol{u}'|^3}$$

where K is a constant depending on a system size. Note that all the physical quantities are converted into macroscopic ones by a course-graining. It is analytically shown that the diffusion fluxes $-\mathbf{D}_s\cdot\nabla\omega_\pm \pm \boldsymbol{V}_s\omega_\pm$ have several physically good properties. For example, the term conserves total system energy. Numerical simulations on PEZY-SC reveal that the key mechanism for self-organization (condensation of same-sign vortices) is provided by the drift term.

(ii) Two-body correlation function

To understand the hole formation around a strong vortices in NNP experiments [6,7], we employ an linear response theory to obtain a 2-body correlation function. Let us consider a single-species (single-sign) point vortex system. Equilibrium stream function with a mean field approximation is given by the following Poisson equation:

$$-\Delta\psi_{eq}(\boldsymbol{r}) = \frac{N\Omega e^{-\beta\Omega\psi_{eq}(\boldsymbol{r})}}{\int e^{-\beta\Omega\psi_{eq}(\boldsymbol{r}')}d\boldsymbol{r}'}$$

By adding a perturbation $c\delta(\boldsymbol{r})$ in the left hand side of the above equation, perturbed equation is obtained

$$-\Delta\psi_c(\boldsymbol{r}) = \frac{N\Omega e^{-\beta\Omega\psi_c(\boldsymbol{r})}}{\int e^{-\beta\Omega\psi_c(\boldsymbol{r}')}d\boldsymbol{r}'} + c\delta(\boldsymbol{r})$$

The solution gives fluctuations of the stream function and vorticity.

$$\delta\psi = \psi_c - \psi_{eq}, \quad \delta\omega = \omega_c - \omega_{eq}$$

By using Ornstein-Zernike relation, correlation function is obtained explicitly. In addition to the details of the analytical result, we will present numerically examined results in the talk.

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