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Robust regression method for LHD charge exchange spectroscopy data with heteroscedastic noise

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1 Introduction

Spatial distributions of the ion temperature and flow in magnetic fusion devices are essential for the transport study. In particular, their space derivatives are also necessary for many transport analyses [1]. Therefore, it is necessary to make smooth regression curves to the observed distributions with noise.

One of the most widespread diagnostics to measure the ion temperature and flow distributions is charge exchange spectroscopy (CXS) [2]. In CXS measurement, their quantities are evaluated from the spectral shapes of the emission during charge exchange interactions between injected neutral particles and fully ionized impurities. This principle results in complexly distributed noise which does not only differ from Gaussian, but also depends on other variables. For example, noise amplitude in the temperature data evaluated from the spectral shape of the emission depends on the spectral intensity. In the case of extremely low spectral intensity, an outlier is often measured. Measurement data with such an outlier cannot be performed a regression analysis accurately with conventional least squares method. Because of the above noise property, fully automatic regression of the CXS data, which is routinely obtained, has been difficult and some human supervision has been always necessary.

In this work, we propose a robust regression method with which the complexly distributed noise is also estimated from vast measurement data simultaneously. We apply our method to the CXS data measured for LHD.

2 Proposed method

We consider the i-th spatial distribution data of the ion temperature $T^{(i)}$ as a sum of the true latent function $f^{(i)}$ and the noise $\epsilon^{(i)}$

$$T^{(i)}(r_j) = f^{(i)}(r_j) + \epsilon^{(i)}(r_j) \tag{1}$$

where r_j is the position of the j –th measurement point in the minor radius r coordinates. We model $f^{(i)}$ as a linear sum of K basis functions

$$f^{(i)}(r_j) = \sum_{k=1}^{K} w_k^{(i)} \phi_k(r_j, \mu_k, \omega_k) \tag{2}$$

where $\phi_k(r_j, \mu_k, \omega_k)$ is a Gaussian basis function with mean μ_k and width ω_k, and $w_k^{(i)}$ is its coefficient. ϕ_k is common to all the spatial distribution data of the ion temperature, while $w_k^{(i)}$ depends on the data index i.

The noise amplitude in the CXS data often does not follow Gaussian distribution. Therefore, we model the noise $\epsilon^{(i)}$ as following Student's t-distribution with mean 0, scale parameter σ and degree of freedom ν

$$\epsilon^{(i)}(r_j) \sim St\Big(0, \sigma\big(f^{(i)}(r_j), V^{(i)}(r_j), I^{(i)}(r_j), \Theta_\sigma\big), \nu\big(f^{(i)}(r_j), V^{(i)}(r_j), I^{(i)}(r_j), \Theta_\nu\big)\Big) \tag{3}$$

where $V^{(i)}$ and $I^{(i)}$ are respectively the spatial distribution data of the flow and spectral intensity measured by CXS simultaneously with $T^{(i)}$. Both σ and ν are functions of $f^{(i)}, V^{(i)}$ and $I^{(i)}$, which contribute to the spectral shapes. For the functions σ and ν, we adopted neural network with parameters Θ_σ and Θ_ν respectively.

Parameters Θ_σ and Θ_ν may include the noise property of the hardware. Therefore, these parameters should be common to all the data. We consider 944 sets of CXS data measured for LHD and optimize all the parameters simultaneously.

3 Result

Fig. 1 shows an example of the temperature distribution data. Black circles are the measurement data. Red curve shows the regression curve with the least squares method, while blue curve shows the regression curve with our method. Blue region shows the 95% interval of the noise distribution estimated with our method.

In Fig. 1, the regression curve with the least squares method overfits the outliers at $r \approx 0.67$ because of the inaccurate assumption on the noise, i.e. it assumes homogeneous Gaussian [3]. On the other hand, the regression curve with our method doesn't overfit the outliers. In addition, the 95% interval of the estimated noise distribution in the outliers is much larger than that in other data.

References

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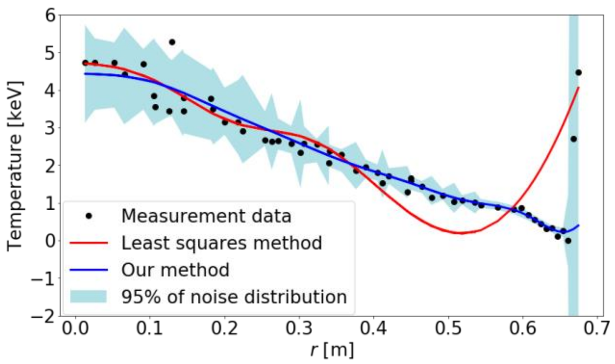

Fig. 1 An example of the ion temperature data, regression curves with least squares method and with our method, and the estimated noise distribution.