F-I23 AAPPS-DPP2020

4th Asia-Pacific Conference on Plasma Physics, 26-31Oct, 2020, Remote e-conference

Why is it so hard to generate 3D MHD equilibria with smoothly nested flux surfaces?

D. Pfefferlé[1]

[1] School of Physics, Mathematics and Computing, The University of Western Australia

e-mail (speaker): david.pfefferle@uwa.edu.au

Renewed interest in stellarator design has sparked questions on the existence and accessibility of three-dimensional magneto-hydrodynamic (MHD) equilibria with "good" nested flux-surfaces. Several numerical tools exist to obtain three-dimensional MHD equilibria. These methods aspire to produce and optimise the magnetic fields so that the field-lines lie on toroidally nested flux-surfaces, which is the basis of plasma confinement in magnetic fusion devices such as tokamaks and stellarators. Under the assumption of translational and/or rotational symmetry (isometries), the MHD equilibrium problem reduces to a two-dimensional elliptic PDE for the scalar flux-function, called the Grad–Shafranov equation. Flux-surfaces naturally correspond to the level sets of the flux-function extruded to surfaces along the direction of symmetry.

It can be proven, whenever the plasma current is not everywhere parallel to the magnetic field, that field-lines lie on nested tori [1,Theorem 7]. However, the embedding of these topological tori may be extremely complicated (knotted) in real space. The theorem also does not inform on the existence nor on the accessibility of such equilibrium configurations. Three-dimensional MHD equilibria (within the class of smooth magnetic fields) whose field-lines lie on simple nested flux-surfaces are thus rather exceptional, fine-tuned solutions.

The goal of this talk is to illustrate an essential difficulty in smoothly deforming an initial configuration with nested flux-surfaces through a family of MHD equilibria to reach a target three-dimensional configuration with equivalent flux-surfaces. The issue from a physical point of view is that flux-surfaces with periodic field-lines (rational rotational transform) are sensitive to resonant perturbations. Whether resonance can be avoided in order to form a smooth sequence of MHD equilibrium states is an interesting line of inquiry, closely related to the study of stationary solutions to Euler equations and their properties.

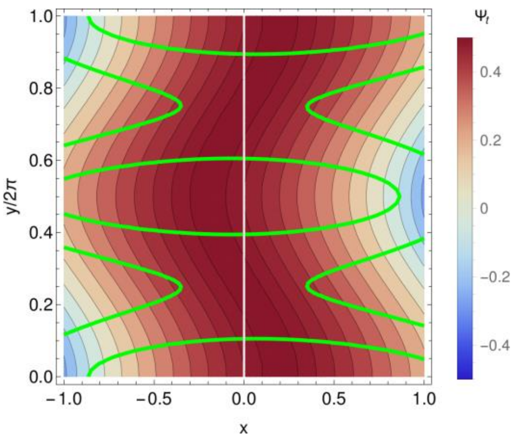

Figure 1. Advected flux from an initial Hahm-Kulsrud-Taylor configuration [2]. The colour contours represent the level sets of the flux-function and the thick green curves are level sets of the current density. The fact that the two do not overlap indicates lack of force balance, occurring at all orders with respect to the boundary deformation [3].

References

[1] V. I. Arnold, Annales de l'Institut Fourier **10**, 137 (1967).

[2] T. S. Hahm and R. M. Kulsrud, The Physics of Fluids **28**, 2412 (1985).

[2] D. Pfefferlé *et al.*, PPCF **62**, 074004 (2020).