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The Correct Cutoff Variable for Coulomb Collision in Plasma

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Based on a long-distance Coulomb interaction of charged particles in the potential $U(r) = \kappa/r$, the plasma kinetic equations always meet the divergence because Rutherford differential cross section

$$\sigma(g,\theta) = \frac{\kappa^2}{4\mu^2 g^4 sin^4(\theta/2)} = \frac{\kappa^2}{(m\Delta\mathrm{v})^4}$$

has a singularity at $m\Delta\mathrm{v} = 0$ here μ is reduced mass and g is the relative velocity of charged particles. Usually, a cutoff variable should be introduced in order to remove the singularity. The traditional way is to make a cutoff either on impact parameter b [1] or scattering angle θ [2]. A third cutoff variable $\Delta\mathrm{v}$ was introduced for removing the singularity [3] [4].

This presentation will compare the differences of the three kinds of cutoff variables, including impact parameter b, scattering angle θ and velocity change $\Delta\mathrm{v}$. It is shown that the singularity at $\Delta\mathrm{v} = 0$ cannot be removed by a cutoff on small scattering angle $\theta(\theta \le \theta_{min})$ unless the relative velocity g is constant. However, in plasma physics, g can vary from zero to infinite due to varied field particle velocity v_F even if the test particle velocity v is a constant. Obviously, the singularity still exists at $g = 0$ after the cutoff on θ_{min} made. The cutoff on scattering angle $\theta \le \theta_{min}$ can not remove the weak collision events with both smaller g and larger θ. In fact, scattering angle θ has already been proved mathematically to be an incorrect cutoff variable [5]. Similarly, the singularity at $\Delta\mathrm{v} = 0$ cannot be removed by a cutoff on large impact parameter b ($b \ge b_{max}$) unless g is constant. Obviously, the singularity still exists at $g = 0$ after the cutoff on b_{max} made. The cutoff on impact parameter $b \ge b_{max}$ cannot remove the weak collision with smaller g and smaller b.

Recently, we claim the impact parameter b is an incorrect cutoff variable. The traditional practice of making the cutoff on small impact parameter $b \le b_{min}$ is a total mistake. Small impact parameter is not the reason of divergence as Landau once pointed out [2] 'if the exact formulae are used, then there would, of course, be no divergence at small b'. Landau's predication is proved by our exact mathematical calculation [6].

The velocity change $\Delta\mathrm{v}$ is so far the only correct cutoff variable that is mathematically proved [4].

Consider a test particle α in a collection of β particles with a Maxwellian distribution, the nth order Fokker-Planck coefficients are defined as the integral $\langle\Delta\mathrm{v}^n\rangle = \int \Delta\mathrm{v}^n f_0(v_\beta, T_\beta) g\sigma sin\theta d\theta d\varphi d\mathrm{v}_\beta$. With the cutoff on $\Delta\mathrm{v} \ge \Delta\mathrm{v}_{min}$, the exact form for arbitrary order of Fokker-Planck coefficients can be derived as

$$\frac{\left\langle \Delta\mathrm{v}_{\parallel}^{n-2(j+k)} \Delta\mathrm{v}_{\perp 1}^{2j} \Delta\mathrm{v}_{\perp 2}^{2k} \right\rangle}{\nu(av_{th})^n} = \sum_{i=0}^{j+k} \frac{(j-1/2)!\,(k-1/2)!\, q_{n+3}^{(n-2i)}(y_{min},u)}{(-1)^{n+j+k+i} i!\,(j+k-i)!}$$

Where the set of functions $q_n^{(k)}$ is defined as

$$q_n^{(k)}(y_{min},u) = \frac{2}{\sqrt{\pi}} \int_{y_{min}}^{\infty} \int_{-1}^{1} e^{-(y+xu)^2} y^n (-x)^k dx dy$$

with $\nu = n_\beta v_{th} \pi (m_\beta k / 2\mu k_B T_\beta)^2$, $y_{min} = \Delta\mathrm{v}_{min}/(av_{th})$, $a = 2\pi/m_\alpha$.

The energy transfer moments are defined as

$$\langle\epsilon^n\rangle = \int \epsilon^n f_0(\mathbf{v}_\beta, T_\beta) g\sigma sin\theta d\theta d\varphi d\mathbf{v}_\beta$$

where $\epsilon = (1/2) m_\alpha (v_\alpha'^2 - v_\alpha^2)$ for a test particle.

By using the cutoff $\Delta\mathrm{v} \ge \Delta\mathrm{v}_{min}$, the nth order of the transfer moments can be derived as

$$\frac{\langle\epsilon^n\rangle}{\nu(m_\alpha a^2 v_{th}^2/2)^n} = \sum_{i=0}^{n} C_n^i \left(\frac{-2u}{a}\right)^i q_{2n+3-i}^{(i)}(y_{min},u)$$

where C_n^i is the binomial coefficient.

The energy equilibrium time and Coulomb logarithm are defined and based on the energy transfer rate,

$$\langle\epsilon\rangle = \int \epsilon f_0(\mathbf{v}_\alpha, T_\alpha) f_0(\mathbf{v}_\beta, T_\beta) g\sigma sin\theta d\theta d\varphi d\mathbf{v}_\beta d\mathbf{v}_\alpha$$

The arbitrary high order of energy transfer rate can be derived by the cutoff $\Delta\mathrm{v} \ge \Delta\mathrm{v}_{min}$ as

$$\{\epsilon^n\} = \int \langle\epsilon^n\rangle f_0(\mathbf{v}_\alpha, T_\alpha) d\mathbf{v}_\alpha$$

$$= \omega\bar{\epsilon}^n \sum_{\boldsymbol{i=0}}^{\left(\frac{n}{2}\right)} \frac{n!\,\Gamma\left(\frac{\gamma+1}{2} + n - i, \frac{\Delta\mathrm{v}_{min}}{k_B T_{eff}}\right)}{i!\,(n-2i)!\,\mathrm{A}^{2i}}$$

Where $\omega = n_\alpha n_\beta v_{av} \pi (m_\beta k / 2\mu k_B T_\beta)^2$, $\bar{\epsilon} = 4\mu^2 k_B (T_\beta - T_\alpha)/m_\alpha m_\beta$.

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