Turbulence model reduction by deep learning

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Anomalous transport, structure formation, and virtually all other aspects of drift-wave turbulence dynamics are encoded in turbulent fluxes which are generated by cross-correlations between fluctuating quantities. Computing these cross-correlations may be considered the central problem of turbulence modeling. However, doing so is a notorious challenge that always requires the use of successive — and sometimes questionable — approximations, such as the assumption of a small parameter, a closure for higher-order moments, or the imposition of an ad-hoc mathematical model.

In this work, we circumvent these difficulties by introducing a novel **data-driven** approach to determine the dependencies of the cross-correlations. The new method uses **deep supervised learning** to infer a mean-field model from numerical solution and/or experiment. This approach, a form of nonparametric regression, leverages deep learning's resilience to the large amounts of noise inherent to turbulence, as well as its ability to model arbitrary nonlinear, multivariate functions.

As a test of concept, we numerically solve the 2-D Hasegawa-Wakatani system, and use computed mean field quantities to train a deep neural network which outputs the local turbulent particle flux and Reynolds stress as a function of local mean gradients, flow properties, and turbulence intensity. The deep neural network detects a previously unreported, non-diffusive particle flux which is proportional to the gradient of vorticity. We recover this flux with a simple analytic calculation. Notably, the deep neural network finds that, in this system, the vorticity gradient effect on the particle flux is much stronger than the direct effect of the local vorticity (shear).

The non-diffusive flux originates from a shift in the drift-wave frequency induced by the nonlinear convection of vorticity. It has immediate implications for structure formation, as it tends to modulate the density profile in the presence of a quasiperiodic zonal flow, forming a staircase. This mechanism for staircase formation is independent from previous models based upon bistability. We show that the same shift in the drift-wave frequency also impacts the local linear growth rate, resulting in corrugations in the intensity profile localized where the density and vorticity gradients have the same sign.

Using the new method, we also uncover a Cahn-Hilliard-type model for the generation of zonal flow via Reynolds stress, which agrees with previous theoretical work. Together with the particle flux, we thus obtain a reduced 1-D model for the turbulent dynamics directly from numerical data. We solve this numerically and compare to direct numerical simulation of the full 2-D system. However, the underlying assumption of a mean-field model is only valid in the weak turbulence regime and breaks down when the dynamics become sufficiently intermittent.

We elucidate the critical role of symmetry to the new method and discuss the method's portability to other applications, such as gyrokinetic simulation or experimental data.