

Time-dependent Probability Density Functions and Information Geometry in the Fusion Low-to-High Confinement Transition

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The need for a proper statistical theory for understanding fusion plasmas has grown significantly, with experiments and simulations revealing ample evidence for non-Gaussian fluctuations, anomalous transport, or intermittency. The latter question the validity of the mean-field-type theory based on small Gaussian fluctuations, necessitating the calculation of an entire probability density function (PDF).

In this paper, we show the importance of intermittency and time-dependent PDF approach in the Low-to-High confinement mode (L-H) transition. To this end, we extend the previous prey-predator-type L-H transition model [1] to a stochastic model by including the two independent, short-correlated Gaussian noises with the strength D_x and D_v for turbulence x ($x^2 = \mathcal{E}$ in [1]) and zonal flow v , respectively. We solve the following time-dependent Fokker-Planck equation for the joint PDF $p(x, v, t)$

$$\frac{\partial p}{\partial t} = -\frac{\partial}{\partial v}(gp) - \frac{\partial}{\partial x}(fp) + D_x \frac{\partial^2 p}{\partial x^2} + D_v \frac{\partial^2 p}{\partial v^2}$$

where f and g are given in [2]. For the results below, $D_v = 10^{-4}$. Figure 1(a) shows the averages $\langle x \rangle$ (solid lines) and $\langle v \rangle$ (dashed lines) against time; (b) shows the standard deviations of x (solid) and v (dashed). [black,blue,red] correspond to $D_x = [1, 4, 16] \cdot 10^{-4}$, respectively. Following the abrupt increase in $\langle v \rangle$ at $t \approx 11$ for all D_x , the dithering I-phase starts where $\langle x \rangle$ and $\langle v \rangle$ oscillate. The dithering phase ends when $\langle x \rangle$ and $\langle v \rangle$ both collapse back towards zero, corresponding to the transition to the H-mode.

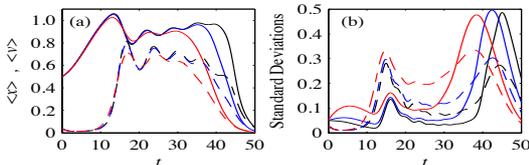


Fig 1: Time trace of mean and standard deviation.

Figure 2 shows time snapshots of joint PDF $p(x, v, t)$ in $x - v$ plane. From top to bottom the three rows (a,b,c) are $D_x = [1,4,16] \cdot 10^{-4}$. In each row, the six panels (1-6) are at times $t = 5, 10, 20, 30, 40, 50$, respectively. $p(x, v, t)$ reveals striking feature including strongly non-Gaussian features and multiple peaks. The final collapse to $x, v \rightarrow 0$ does not consist of a simple motion of the peak toward the origin. Instead, comparing times $t = 30, 40, 50$, we see how the original peak remains largely in the same position while a secondary peak grows and eventually dominates near the origin even when $\langle x \rangle$ and $\langle v \rangle$ continuously decrease to zero in time in figure 1.

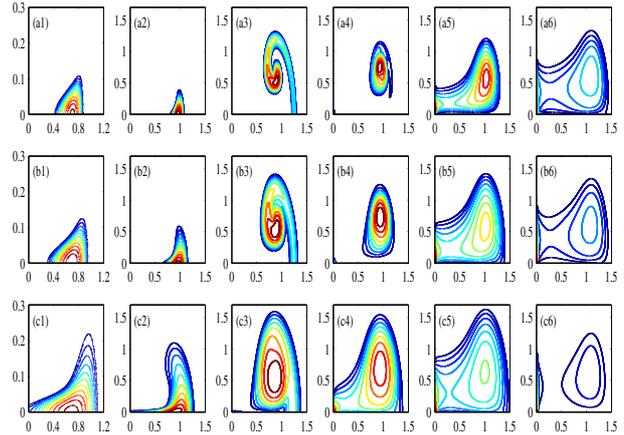


Fig 2: Snapshots of joint PDF $p(x, v, t)$ in $x - v$ plane

Figures 3-4 show the marginal PDFs $p(x, t)$ and $p(v, t)$ at $t = 10, 20, 30, 40$ in (a,b,c,d), respectively (using the same colour coding as in figure 1). We see the strong deviations from Gaussian PDF and a significant asymmetry around the peak and bimodal nature with the number of peaks changing in time, highlighting the limited utility of mean value and variance. Furthermore, $p(v, t)$ is notably more stretched than $p(x, t)$ at the right tail; rare events of large v are more common than rare events of large x , suggesting that the transitions to I-phase and H-mode are facilitated by rare events of strong zonal flow v . That is, intermittency of zonal flows can play an important role in promoting the L-H transition. We also discuss a novel information geometric method [3] by using information length and show their utility in forecasting transitions and self-regulation between turbulence and zonal flow.

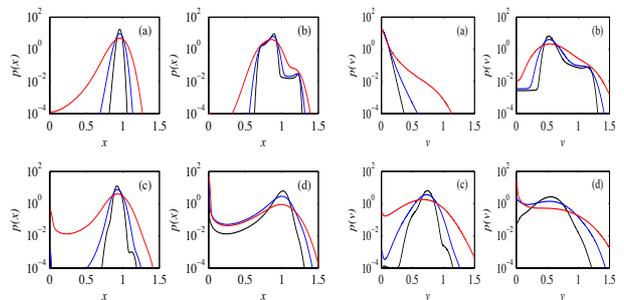


Fig 3: $p(x, t)$

Fig 4: $p(v, t)$

References

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