

## First-principles formulation of resonance broadened quasilinear theory near an instability threshold

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The collisional broadening of resonance lines is a universal phenomenon in physics. For example, in atomic physics, collisions lead to abrupt changes in the phase and plane of vibration, thereby destroying phase coherence and leading to uncertainty in the associated photon energy. This leads to broadening of an atom emission/absorption profile [1,2]. In plasma physics, decoherence of the orbital motion of resonant particles allows the reduction of reversible equations of motion into a diffusive system of equations that governs the resonant particle dynamics without detailed tracking of the ballistic motion—as is the case in the widely used quasilinear (QL) kinetic plasma formulation. This work shows how to calculate, from first principles, the resonance function i.e., the envelope function that weights the strength of the resonant interaction, in QL theory in the presence of collisions. It is demonstrated that a quasilinear system that employs the calculated broadening functions (Fig. 1a) recently reported [3] automatically recovers the nonlinear growth rate and mode saturation levels for near-threshold plasmas previously calculated from a significantly more complex nonlinear kinetic theory based on solving a time delayed integro-differential equation [4]. Moreover, it is shown that a QL theory can be constructed for a single resonance, provided that it experiences enough background stochasticity. The results enable realistic reduced modeling of diffusive transport observed in fusion devices.

A systematic methodology for establishing a transport theory is developed from first principles, which can be of interest over a range of disciplines that employ kinetic theory. The present formulation gives simple analytical answers to the shape of discrete-resonance collisional functions in quasilinear theory, for both Krook and Fokker-Planck scattering collisions. These functions, that integrate to unity, replace a simple delta function that appears in the diffusion coefficient for the case of no broadening. It is shown that the knowledge of these functions removes a major arbitrariness with respect to previous resonance broadening approaches, which consisted of tuning broadening parameters to match the expected saturation levels [5]. In addition, analytical forms for the modification of the distribution function (shown in Fig. 1b) can be naturally constructed. These forms are particularly useful for code verification akin to studies reported in [6].

This formulation is required to model and verify the fast ion relaxation in tokamaks upon interaction with Alfvénic eigenmodes being implemented in the Resonance Broadened Quasilinear (RBQ) code [7],

where particle diffusion occurs along the canonical toroidal momentum and the energy of the beam ion distribution function.

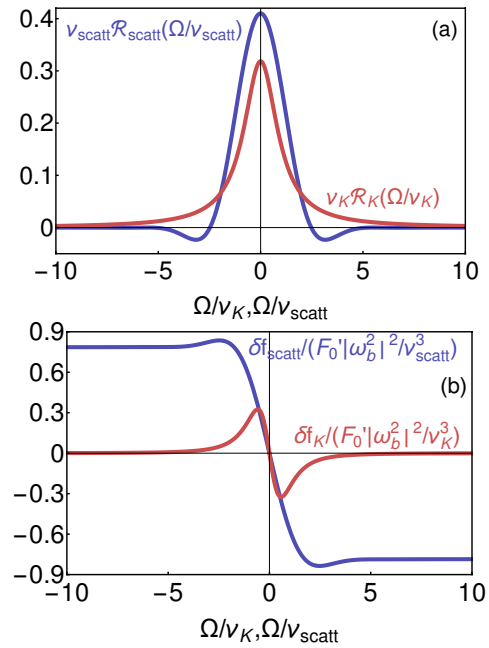


Figure 1: (a) Resonance function and (b) the modification of the distribution function around the center of a resonance vs normalized frequency variable. The red and blue curves correspond to the Krook and scattering cases, respectively.

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