Fractality and cumulative entropy of a magnetized plasma driven by fractional Brownian motion

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Shannon entropy of a probability distribution can be estimated directly by knowing the probabilities p_i of each state i. In the case of real time series, p_i can be derived from a binning of the possible values, a strategy that can give a bad estimate of S if the number of intervals is not properly chosen. To overcome this issue, Di Crescenzo et al. proposed an entropy based on the cumulative probability distribution (CDF), defined as

$$\mathrm{CE} = -\int F(x) \ln F(x)\, dx \,,$$

where $F(x)$ is the CDF of a random variable x. Recently [3,4], the fractality of a magnetohydrodynamic turbulence shell model has been studied. In particular, the GOY-type shell model described by the evolution equations for the velocity $u_n(t)$ and magnetic field $b_n(t)$ fluctuations

$$\begin{aligned}\dot{u}_n &= -\nu k_n^2 u_n + i k_n\{(u_{n+1}u_{n+2} - b_{n+1}b_{n+2}) \\ &\quad -(u_{n-1}u_{n+1} - b_{n-1}b_{n+1})/4 \\ &\quad -(u_{n-2}u_{n-1} - b_{n-2}b_{n-1})/8\}^* + f_n \,, \\ \dot{b}_n &= -\eta k_n^2 b_n + i k_n\{(u_{n+1}b_{n+2} - b_{n+1}u_{n+2}) \\ &\quad +(u_{n-1}b_{n+1} - b_{n-1}u_{n+1}) \\ &\quad +(u_{n-2}b_{n-1} - b_{n-2}u_{n-1})\}^*/6 + g_n \,.\end{aligned}$$

corresponding to the eddy's scale of length $l \sim k_n^{-1}$. The cumulative entropy of the dissipated magnetic energy b is calculated to analyze the behavior of the system regarding its intrinsic non-linearity. To solve the model, we only need to specify the values of the kinematic viscosity ν, resistivity μ and number of shells. In this case, we consider $\nu = \eta = 10^{-4}$ and $N = 19$ shells and integrate numerically the evolution equations. In Ref. [4], the forcing terms for the velocity and the magnetic field, f_n and g_n, were obtained from solar wind velocity and magnetic field data, representing the evolution of the Earth's magnetosphere under various levels of intermittency of solar wind forcing (as measured by its fractal dimension). Here, to systematically study this issue, the forcing is given by a fractional Brownian motion time series with various Hurst exponents. Thus, we investigate the possible correlations between the cumulative entropy of the dissipated magnetic energy time series driven by the fractal time series.

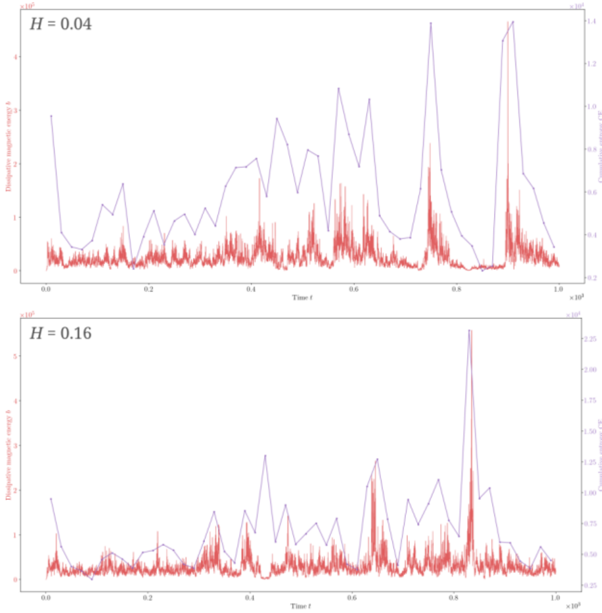

Figure 1. Cumulative entropy of a magnetized plasma forced with Hurst parameter $H = 0.04$ (top) and $H = 0.16$ (bottom) compared with its dissipative magnetic energy.

A correlation between the cumulative entropy and the dissipative activity of the system can be observed in the preliminary results. Furthermore, a sensitivity to fractal forcing induced by time series with different Hurst parameters is suggested.

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References

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