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Statistical Physics and Order in Euler Turbulence

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Incompressible Euler equation has zero viscosity and no external forcing, hence, it can be treated as an isolated system. Lee and Kraichnan argued that Euler turbulence attains equilibrium with zero energy flux. It has been shown that three-dimensional Euler turbulence asymptotically approaches this equilibrium state. In this presentation, we show that two-dimensional (2D) Euler turbulence is out of equilibrium, and it exhibits evolution from disorder to order, even though the system is an isolated one [1,2].

We performed a pseudo-spectral simulation of 2D Euler turbulence on a 512^2 grid with initial velocity shown in Fig. 1(top). We observe that the system evolves to a more ordered state of Fig. 1(bottom). The small wavenumber modes exhibit an inverse energy cascade, hence, *detailed balance of energy transfer* is broken. The energy spectrum of the flow too deviates from the equilibrium formula predicted by Kraichnan. Similar features are observed for other initial states as well. Thus, we show that the 2D Euler turbulence is out of equilibrium [3].

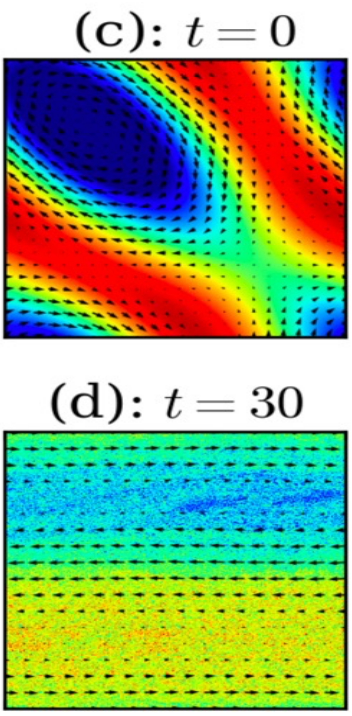

Fig. 1: The initial state (top figure) and final state (bottom figure) of a 2D Euler turbulence simulation.

Euler flow has zero dissipation. Hence its thermodynamic entropy is constant, and it cannot capture the variable order of the flow,. Therefore, we propose "hydrodynamic entropy" for describing the disorder in Euler turbulence. We define the hydrodynamic entropy as $-\sum_{\boldsymbol{k}} p_k \, log(p_k)$, where $p_k = E(\boldsymbol{k})/E$. Here, $E(\boldsymbol{k})$ is the modal energy, and E is the total energy. As shown in Fig. 2, the hydrodynamic entropy decreases with time for a significant period. Hence, ***2D Euler turbulence is a unique isolated system that exhibits evolution from disorder to order* [3].**

We believe that the above findings will have a strong impact in other areas of physics, e.g., in nonequilibrium statistical physics, self-gravitating systems, and MHD and plasma turbulence, and arrow of time [4]. In this talk will review the status of Euler turbulence and its connections to statistical physics.

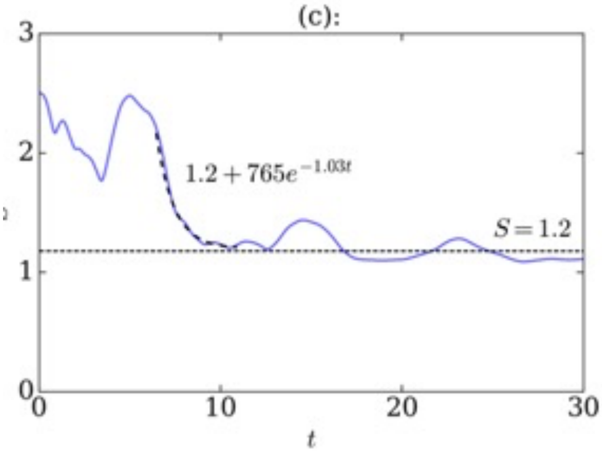

Figure 2: A plot of the time series of hydrodynamic entropy of the above run. Clearly, the final state is more ordered than the initial one.

References

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