

Duct effect of magnetic structures on whistler waves

Xiongdong Yu¹ and Zhigang Yuan^{1*}

¹School of Electronic Information, Wuhan University, Wuhan, China

e-mail (speaker): yuxiongdong@whu.edu.cn

Theoretical Modeling: For simplicity, we consider the circumstance where the background magnetic field is along the z-axis and the field intensity only varies along the x-axis. Thus, each wave field component (Q) has the form (Swanson, 2003)

$$Q = Q(x) \exp\left(i\frac{\omega}{c}pz\right) \exp\left[i\frac{\omega}{c}\int q(x)dx\right]$$

where $p = ck/\omega$ and $q(x) = ck_{\perp}(x)/\omega$ are the parallel and perpendicular refractive indices, respectively, with ω denoting the wave frequency and c denoting the speed of light. Note that the wave normal angle (θ) is determined by p and q as $\tan\theta = q/p$.

Under the cold plasma approximation and in the frequency range of whistler waves, one can solve the solutions for $q(x)$ as (e.g., Karpman and Kaufman, 1982)

$$q_j^2 = -\frac{\alpha}{u^2} - \left(1 - \frac{1}{2u^2}\right)p^2 + (-1)^j \frac{p}{2u^2} \sqrt{p^2 - 4\alpha}$$

To investigate the effect of magnetic peaks and dips, we use the following model for the magnetic field:

$$\vec{B} = \hat{z}B_z = \hat{z}B_{z0} \left[1 + A \tanh^2\left(\frac{x}{D}\right)\right]$$

where $\tanh^2(\cdot)$ is the square of the hyperbolic tangent function used to model the variation in the magnetic structures; B_{z0} is the magnetic intensity at the center of the magnetic field structure ($x = 0$); D is the scale of the inhomogeneity, which is assumed to be sufficiently large compared with the wavelength of interest that the tunneling transformation is negligibly weak; and A is a constant parameter used to control the type of magnetic structures.

Theoretical Results: Figure 1 shows the calculated results for whistler waves with different wave properties (u and p) in a magnetic peak, where the left column shows lower-band ($u < 0.5$) whistler waves and the right column shows upper-band waves ($u > 0.5$). The parameters for the magnetic peak ($B_{z0} = 135$ nT, $A = -0.04$) and the plasma condition ($\alpha(0) = 12$, i.e., α at the center $x = 0$) are used here. In Panels (c) and (d), the red curves correspond to the branches for $\pm q_2$ and the blue curves correspond to $\pm q_1$ (when extant). The gray rectangles in Panels (a) and (b) indicate the wave-trapped regions. Figure 1 clearly shows that both lower- and upper-band whistler waves can be trapped in the magnetic peak but through different wave couplings. For the lower-band whistler waves, trapping is achieved by wave couplings $\pm q_1 \leftrightarrow \pm q_2$ at positions $q_1 = q_2$. In the trapped region around the center of the magnetic peak, both quasi-parallel ($|\theta| < 45^\circ$) and oblique ($|\theta| > 45^\circ$) lower-band whistler waves can be found, as shown in Panels (a) and (c). By contrast, only quasi-parallel emissions can be trapped for upper-band whistler waves in a magnetic peak through wave couplings $q_2 \leftrightarrow -q_2$ at positions where $q_2(x) = 0$.

The calculated results for whistler waves in a magnetic dip

are shown in Figure 2 in the same format used in Figure 1. Here, a magnetic dip with $B_{z0} = 185$ nT and $A = 0.23$, as well as $\alpha(0) = 9$, are utilized. Figure 2 clearly shows that regardless of the wave frequencies, oblique whistler waves (viz., the branches $\pm q_2$, as $q_2 \gg q_1$) will propagate across the magnetic dip (Panel c) or be reflected at some points before reaching the center (Panel d). Quasi-parallel lower-band whistler waves ($\pm q_1$ in Panel c) can be trapped through wave couplings ($q_1 \leftrightarrow -q_1$) at $q_1(x) = 0$. In such cases, whistler waves are ducted in the regions around the center of the magnetic dip (Panel a) at small normal angles (up to 36° in this case).

Summary: In conclusion, we have used theoretical analysis and satellite observations to investigate the duct effect of magnetic peaks and dips on whistler waves. Although a simple model has been used to describe magnetic structures, our theoretical analysis shows ducting propagation is possible for whistler waves in both magnetic peaks and dips. The trapping conditions under different circumstances are also presented but should be adjusted when different magnetic field models are used.

References

- Swanson (2003). Plasma waves. Institute of Physics Publishing.
 Karpman, V. I., & Kaufman, R. N. (1982). Whistler wave propagation in density ducts. Journal of Plasma Physics (27), part 2, 225–238.

Figure 1

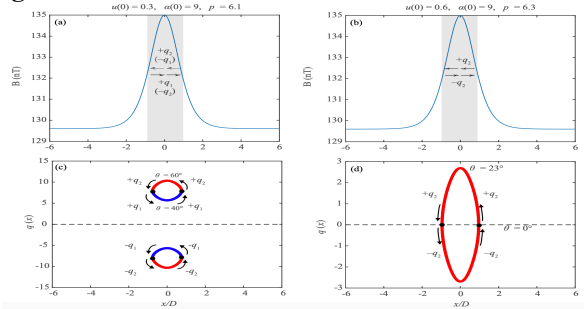


Figure 2

