

## Nonlinear phase dynamics in a chirping wave

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In fusion devices, phase dynamics of particles interacting with waves plays a substantial role in the evolution of wave amplitude and frequency. In the problem of non-linear frequency chirping/sweeping waves, which occur frequently in fusion experiments, the motion of these particles is governed by a time dependent Hamiltonian (see Ref. [1]) where adiabatic conservation laws can be found provided that the system evolves slowly. During the evolution of the wave frequency, the existence of adiabatic invariants for trapped particles in the wave potential together with Liouville's theorem imply the existence of nested co-centric layers of phase-space density i.e. level sets of the distribution function in phase-space, a stepped distribution profile. This enables using phase-space waterbags, a Lagrangian contour approach, to calculate the perturbed density of particles during chirping. In Refs. [2-7] and Refs. [8-10], the fast time-dependency of the Hamiltonian is canceled using a canonical transformation to a moving frame and subsequently the lowest order adiabatic invariant has been used. The former transfers the dynamics to a non-inertial frame where the lowest-order adiabatic invariant contains the terms corresponding to the fictitious force, while the latter considers a frame that only moves with the fast oscillatory component of the wave and hence the corresponding term appears to the next order correction. In this work, we investigate and compare the accuracy of the adiabatic invariants used in these models as a function of frequency chirping rate. We also develop a simulation model to evaluate the phase-space, shown in Fig.1, using the wave parameters reported in the theoretical water-bag model of Ref. [7] using phase-space waterbags.

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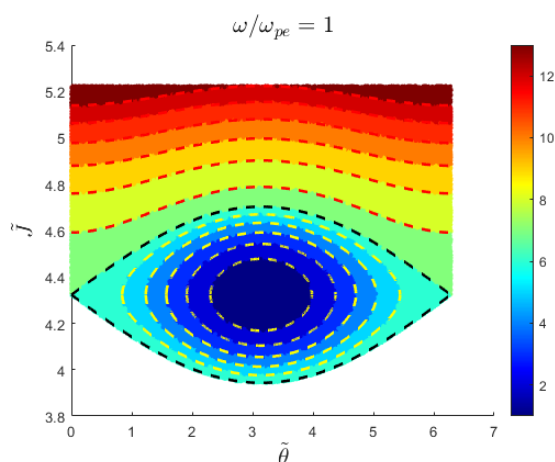


Figure 1. Initial phase-space of energetic particles during the evolution of a chirping wave. The color bar denotes the phase-space density of fast particles discretized using layers of adiabatic invariants. Each level set of the phase-space density denotes a phase-space waterbag i.e. Lagrangian contour approach. The area inside the separatrix (black dashed line) shows co-centric layers of phase-space density.

### References

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