6th Asia-Pacific Conference on Plasma Physics, 9-14 Oct, 2021, Remote e-conference

The General Metriplectic Formalism for Describing Dissipation and its Computational Uses

Philip J. Morrison[1],

[1] Department of Physics and Institute for Fusion Studies University of Texas at Austin,

e-mail: morrison@physics.utexas.edu

Although an early generalization of Lagrangian mechanics to include dissipation was proposed by Rayleigh (1894) and subsequently various specific frameworks for dissipation have been given, e.g., for phase separation in Cahn-Hilliard (1958) and Ricci flows in Hamilton (1982), here we discuss bracket descriptions for dissipation that were motivated by the noncanonical Poisson bracket formulation of plasma models presented in e.g. [1-3]. Here the motivation was to place dissipation in a kind of bracket formalism that complements the nondissipative Poisson bracket formalism. Early attempts were given in [3-7]. The axioms of the formalism presented in [6,7] were later in [8] called metriplectic dynamics. Subsequent aspects of metriplectic dynamics were described by the author in [9-11].

The vector fields of metriplectic dynamical systems have the form $\{z,F\} + (z,F)$, where z represents the dynamical variable and $F=H + S$ is a free energy functional, composed of an energy H plus entropy S (a Lyapunov functional), that generates the dynamics. Here $\{\ ,\ \}$ is a Poisson bracket while $(\ ,\ )$ is a symmetric bilinear product on phase space functions that generates dissipative terms of the dynamical system. Because Casimir invariants are candidate entropies $\{A,S\}=0$ for all functionals A; similarly, $(A,H)=0$ for all functionals A. These assumptions together with $(S,S)\geq 0$ guarantee that the first and second laws of thermodynamics are satisfied for a dynamical system.

Just as symplectic integrators preserve canonical Hamiltonian structure, with some advantages, there have been recent efforts to preserve noncanonical Hamiltonian structure (see e.g. [12]) and even metriplectic structure [13]. Our recent efforts [14,15] will be detailed.

Another kind of dissipative dynamics, double bracket dynamics, was proposed in [16,17]. Here energy serves as a Lyapunov function with all Casimir invariants being conserved. This formalism was proposed as a method for computation of equilibrium states, that was generalized and elevated to a practical method for fluid systems in [18,19] and plasma equilibria in [20-22].

To summarize, in this talk I will talk about general dissipative structures and their use for designing computational algorithms.

References

[1] P. J. Morrison and J. M. Greene, Phys. Rev. Lett. **45**, 790 (1980)

[2] P. J. Morrison, AIP Conf. Proc. **88**, 13 (1982)

[3] P. J. Morrison and R. D. Hazeltine, Phys. Fluids **27**, 886 (1984)

[4] A. N. Kaufman and P. J. Morrison, Phys. Lett. A **88**, 405 (1982)

[5] A. N. Kaufman, Phys. Lett. A **100**, 419 (1984)

[6] P. J. Morrison, Phys. Lett. A **100**, 423 (1984)

[7] P. J. Morrison, Tech. Rep. PAM–228, University of California at Berkeley (1984)

[8] P. J. Morrison, Physica D **18**, 410 (1986)

[9] P. J. Morrison, J. Physics: Conf. Ser. **169**, 012006 (2009)

[10] M. Materassi and P. J. Morrison, J. Cybernetics and Physics **7**, 78 (2015)

[11] B. Coquinot and P. J. Morrison, J. Plasma Phys. **86**, 835860302 (32pp) (2020)

[12] P. J. Morrison, Phys. Plasmas 24, 055502 (2017)

[13] M. Kraus and E. Hirvijok, Phys. Plasmas **24**, 102311 (2017)

[14] C. Bressan, M. Kraus, P. J. Morrison, and O. Maj, J. Phys.: Conf. Series **1125**, 012002 (2018)

[15] C. Bressan, Ph.D. thesis, Technical University of Munich (2022)

[16] R. W. Brockett, Proc. IEEE **27**, 799 (1988)

[17] G. K. Vallis, G. Carnevale, and W. R. Young, J. Fluid Mech. 207, **133** (1989)

[18] G. R. Flierl and P. J. Morrison, Physica D **240**, 212 (2011)

[19] G. R. Flierl, P. J. Morrison, and R. V. Swaminathan, Fluids: Topical Collection "Geophysical Fluid Dynamics" **4**, 104 (2019)

[20] M. Furukawa and P. J. Morrison, Plasma Phys. Control. Fusion **59**, 054001 (2017)

[21] M. Furukawa, T. Watanabe, P. J. Morrison, and K. Ichiguchi, Phys. Plasmas **25**, 082506 (2018)

[22] M. Furukawa and P. J. Morrison, arXiv:2205.14607v1 [physics.plasm-ph] 29 May (2022)