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A class of particle-based Hamiltonian reductions for the Vlasov equation: What a PIC code is really solving

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The efficacy of the particle-in-cell (PIC) method as a discretization scheme for the Vlasov equation implicitly relies on the fact that the representation of the phase-space distribution in terms of a weighted sum of delta functions constitutes an exact Hamiltonian reduction of the continuous dynamics. For example, this reduction was used to create the GEMPIC [1], a structure preserving algorithm that is an example of a Poisson integrator [2].

In an effort to reduce statistical noise, some form of filtering is frequently added to smooth out the source terms that are needed for the field solvers (e.g. charge and current density). This generally takes the form of convolution with some kernel function. In this talk, we will discuss recent work [3] where we consider, from a general perspective, the incorporation of smoothing into finite-dimensional Hamiltonian reductions of the Vlasov equation and other kinetic and fluid models and the subtle ways that the continuous dynamics must be modified in order to admit a smoothed-particle finite-dimensional reduction. To this end we find the tools of reproducing kernel Hilbert spaces [4] to be particularly useful.

We find that smoothed PIC methods generically are not exact reductions of the Vlasov-Poisson or Vlasov-Maxwell equations, but rather approximations to these models in which the Hamiltonian has been regularized at small scales via a smoothing convolution operator. This work demonstrates exactly how such smoothed PIC methods come from regularized continuum theories, thus providing an analytical tool to study the impacts of smoothing in particle-based discretizations. Of particular note is the manner in which these smoothed continuum theories may be interpreted as Lie-Poisson Hamiltonian field theories built from the inner product structure of a suitably defined reproducing kernel Hilbert space [4].

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References

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