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Density matrix and information entropies in multi-field turbulence

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In recent decades, information entropy has been investigated intensively in various physics fields such as quantum gravity theory, condensed matter physics, and non-equilibrium thermodynamics. In particular, there have been several attempts to understand the dynamics of turbulence with information entropies. Although these studies have revealed some important features of integrated turbulent intensity or time series data of turbulence, the inhomogeneous nature of spatial patterns and their bifurcation are not properly treated. Then, we propose a novel definition of quantum-inspired information entropies for turbulent fields using the multi-field singular value decomposition (MFSVD) [1], which can decompose multiple fields simultaneously retaining the inhomogeneity and the multi-scale nature in the original field quantities.

Multiple fields of spatio-temporally evolving physical quantities $f_k(\boldsymbol{x},t)$ is decomposed by the MFSVD such as $f_k(\boldsymbol{x},t)=\sum_i s_i h_i^{(k)}(t)\psi_i(\boldsymbol{x})$. Here, the spatial structure $\psi_i(\boldsymbol{x})$ is common to all fields, and it is orthonormal. Thus, $\psi_i(\boldsymbol{x})$ can be regarded as a unitary basis of the Hilbert space of the field quantities $|\psi_i\rangle$. The density matrix of the turbulent field is defined from the basis; $\rho \coloneqq \sum_i \eta_i |\psi_i\rangle\langle\psi_i|$ where $\eta_i \coloneqq \left(s_i h_i^{(k)}\right)^2 / \sum_i \left(s_i h_i^{(k)}\right)^2$. This is an analogy of a mixed state in quantum mechanics for turbulent systems. Then, the von Neumann entropy (vNE) S_{vN} is formulated similar to quantum mechanics such as $S_{\mathrm{vN}} \coloneqq -\mathrm{Tr}(\rho \ln \rho) = -\sum_i \eta_i \ln \eta_i$. Since $h_i^{(k)} \sim O(1)$, the vNE is approximately regarded as the Shannon entropy of the SVD mode spectrum s_i.

The Hasegawa-Wakatani equation system [2,3] is utilized to demonstrate the effectiveness of the present information analysis. In this physical model, the drift-wave turbulence and the zonal flow (ZF) that is spontaneously generated from the nonlinear interactions with the turbulence are considered. Depending on physical parameters, the quasi-steady state bifurcates into turbulence-dominated and ZF-dominated as known by previous studies [3]. We carry out numerical simulations of the Hasegawa-Wakatani equation in the two-dimensional rectangular geometry scanning physical parameters. Then, the MFSVD is applied to the time series of field data in quasi-steady states, and the density matrix and the associated vNE for the turbulence and the ZF are analyzed.

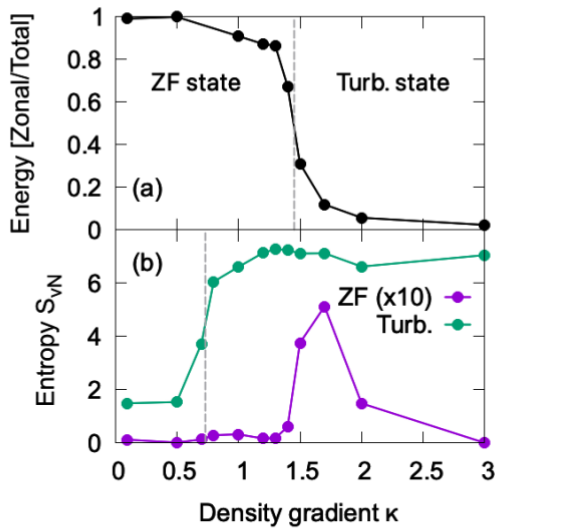

Figure 1. κ dependence of the energy distribution ratio and the von Neumann entropy

Fig. 1(a) shows the dependence on the density gradient κ of the ZF energy ratio. Since the density gradient drives the primary instability, the turbulence strongly grows in the case of large κ. The phase-transition point exists at $\kappa \simeq 1.4$, consistent with previous works. On the other hand, the vNE for the turbulence increases as κ increases and exhibits the phase-transition-like behavior at $\kappa \simeq 0.7$. Thus, it is demonstrated that the quasi-steady states are classified by not only the energy but also the entropy. From the contour plots of the turbulence field (not shown), it is found that the turbulent vortices are trapped by the ZF in the small vNE state. As κ increases, the trapping of the vortices is lost, and the vNE increases, despite the dominance of the ZF.

In this work, a novel feature of turbulent systems is revealed due to the quantum-inspired information entropies. Another aspect of the entropy in turbulent systems, an entanglement entropy for nonlinear interactions between the turbulence, will also be presented.

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