

Mode-Locking, Single- and Double-Well Chaos in Periodically Forced Quantum Degenerate Plasmas: Raveling Unexplored Regimes of the Burgers Paradigm

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Nonlinear coupled drift-ion acoustic waves (NCDIAWs) in quantum degenerate *epi* plasmas with adiabatically trapped electrons hold critical significance for understanding wave dynamics in compact astrophysical objects (e.g., white dwarfs) [1, 2] and laboratory plasmas under extreme cryogenic conditions [3, 4]. Using a mixed kinetic-fluid framework, we derive a nonlinear evolution equation for NCDIAWs characterized by a fractional 3/2-power nonlinearity. The dynamics are investigated through a planar Burgers equation system subjected to external periodic forcing. Employing phase plane theory, Sagdeev potentials, Poincaré sections, Lyapunov exponents, and bifurcation analysis, we uncover novel regimes in the Burgers paradigm, including mode-locking, double-well chaos, and aperiodic transitions under resonant frequency drives.

Key findings demonstrate that the Burgers equation generates shock wave pairs, while the external forcing frequency (ω) governs transitions between ordered and chaotic states. Off-resonant ω stabilizes quasiperiodic motion, whereas near-resonant ω triggers mode-locking/unlocking regimes. Strikingly, frequencies proximal to the resonance region of the system induce double-well chaos, a bifurcation between single- and

double-well chaotic attractors, marking a previously unexplored phenomenon in quantum plasma wave systems. These results bridge theoretical plasma physics with observable instabilities in astrophysical and laboratory environments, offering a pathway to controlling chaos in nonlinear Burgers-type systems.

References

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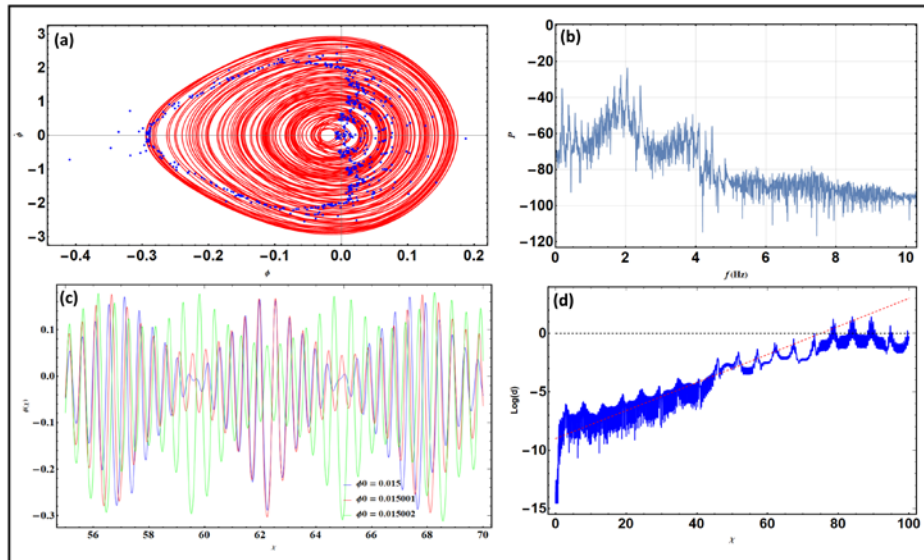


Figure 1: Nonlinear dynamics of the system under an external periodic force frequency $\omega = 2.23$ with amplitude $f_0 = 2.521$ and the other physical parameters are $B_0 = 10^{10}G$, $T = 0.1$, $n_0 = 10^{27}cm^{-3}$, $\beta = 0.1$, $v_{in} = 3 \times 10^{13}s^{-1}$, $\theta = 70^\circ$, and $v_{de} = 0.4$. (a) Phase portrait (red trajectories) overlaid with the Poincaré map (blue dots). (b) Fast Fourier transform of the electrostatic potential ϕ . (c) Time series of ϕ for three closely spaced initial conditions. (d) Lyapunov exponent.