Extracting stochastic model for predator-prey dynamic of turbulence and zonal flow with limited data

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Surrogate models are increasingly vital in physics, such as fluid dynamics, plasma physics, and chaos theory. By leveraging observational data, they reduce computational costs by approximating input–output relationships [1,2]. The predator-prey dynamics between zonal flows and turbulence is a well-known phenomenon in fusion plasmas. However, a direction extraction of a surrogate model from data is difficult, as simulations reveal large and random fluctuations due to the stochastic nature of turbulences (see Figure 1).

We introduce a physics-informed stochastic model (PISM), based on stochastic differential equations (SDEs).[3] The model incorporates grid averaging and the unscented transform (UT)[4], enabling the extraction of an underlying SDE model even with limited data. Our data for training are generated from simulations of the modified Hasegawa-Wakatani equation[5]. Our model takes form as:

$$
\begin{aligned}
dE_T &= g_{11}(E_T, E_Z; \boldsymbol{\theta_{11}})dt + g_{21}(E_T, E_Z; \boldsymbol{\theta_{21}})dw, \\
dE_Z &= g_{12}(E_T, E_Z; \boldsymbol{\theta_{12}})dt - g_{22}(E_T, E_{Z;}; \boldsymbol{\theta_{22}})dw, \quad (1)
\end{aligned}
$$

Where $E_{T,}E_Z$ are energy of zonal flow and energy of turbulence respectively. $\boldsymbol{\theta}$ is parameter of neural network and dw is a Brownian motion. In our model, drift functions g_{11}, g_{12} and diffusion functions g_{21}, g_{22} are learned sequentially. The learned SDE successfully reproduces the predator-prey dynamics[6], including the randomness (Figure 1) with a KL divergence of 0.15 between the data and the prediction. We recover the well-known result that the efficiency of zonal flow shearing effects on turbulences decreases as the zonal flow amplitude increases. Additionally, we find that in the absence of randomness, predator-prey dynamics between turbulence and zonal flows would damp. We also perform a parameter scan over the density gradient length.

References

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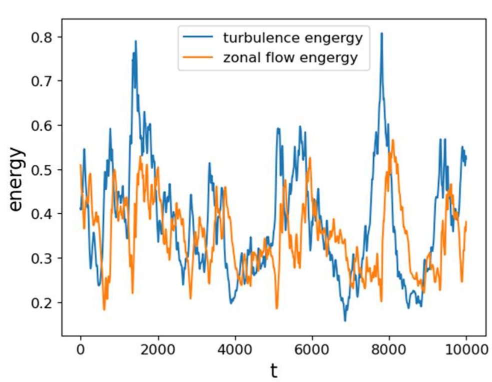

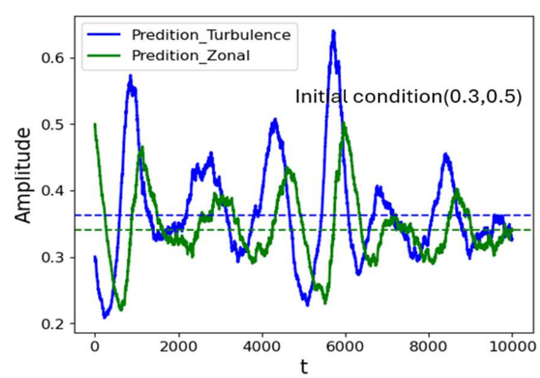

Figure 1 (Left) Turbulence and zonal flow energy from a simulation of the modified Hasegawa-Wakatani equation. (Right) Trajectories generated from the learned stochastic differential equation.