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Numerical Extraction of Nearest Canonical Equilibrium Distribution via Natural Gradient Descent method

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This work presents an efficient method for numerically extracting the nearest canonical equilibrium distribution f_{NE} from any given axisymmetric distribution function of tokamak plasmas. This approach involves formulating the problem as an optimization task based on the discrete form of the gyrokinetic Vlasov equation. An iterative scheme utilizing natural gradient descent is employed with a specified numerical accuracy. Additionally, an enhancement algorithm is incorporated to accelerate the convergence process for phase space points near the trapped-passing boundary. This method has been implemented in the gyrokinetic code NLT and verified against the conventional method based on the orbit average operator. It is found that the numerical accuracy of the new method is significantly higher than that of the direct orbit average method and overcomes the stiff numerical difficulties near trapped-passing boundaries. Possible applications of this algorithm are also discussed.

Introduction: The canonical equilibrium distribution [1] is defined to be constant along the unperturbed orbits:

$$\dot{Z}_0^i \frac{\partial}{\partial Z^i} f = 0 \qquad (1)$$

Here $\mathbf{Z} = (\boldsymbol{X}, v_\parallel, \mu)$ is the gyrocenter coordinate variables, $\dot{Z}_0^i$ is the unperturbed motion determined by equilibrium fields. This work propose an algorithm to extract the "Nearest Canonical Equilibrium Distribution" (abbreviated as 'NE' for convenience) to the given distribution function f_0. This extraction algorithm establishes correct initial conditions for simulations using the full-f method or the direct δf approach [1]. Besides, it facilitates precise equilibrium updating during prolonged δf simulations [2] by reliably isolating the equilibrium component from the turbulent component δf. We also notice that the extracted NE is consistent with the definition of a recent concept called 'Phase Space Zonal Structure' (PSZS) [3], which has drawn a lot of attraction in EP transport problems. In addition, the NE extraction algorithm also performs an effective dimension reduction from 4D function in gyrokinetic coordinate down to the 3D constants of motion (CoM) space (P_ζ, E, μ), which is highly relevant in EP diagnoses [4].

Method: The extraction of f_{NE} is realized by formulating the problem as an optimization task utilizing natural gradient descent method. A positive weight function is multiplied to the loss function that significantly accelerate the convergence rate. Starting from given distribution function f_0, this algorithm is guaranteed to converge to its nearest canonical equilibrium f_{NE}. The algorithm is applied within the 5D gyrokinetic code Nonlinear Lie-Transform (NLT). The time consuming of this algorithm is negligible comparing to the time required for simulating transport phenomena.

Results: This algorithm has been verified to find NE with high accuracy. Fig 1 shows the normalized loss $S_{norm} = \sqrt{\frac{\int \mathcal{J} d^5 Z \left(\dot{z}_0^i \frac{\partial f}{\partial z^i}\right)^2}{\int \mathcal{J} d^5 Z f_0^2}}$ decreasing with iterations. This convergence result indicates that $\dot{Z}_0^i \frac{\partial}{\partial Z^i} f_{NE}$ can be considered exactly zero, validating the effectiveness of the algorithm.

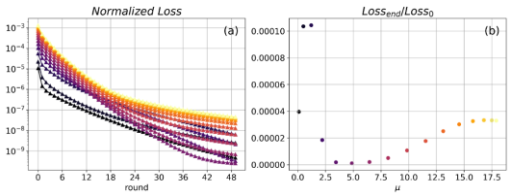

Figure 1: (a) Normalized Loss S_{norm} decreasing trend with iterations. Different colors represent different values of μ (lighter color for larger μ); (b) Relative change in loss.

Besides, the extracted NE can be proven to be consistent with an orbit average, i.e. $f_{NE} = \frac{1}{\tau_b} \oint d\tau \, f_0$. The results are compared to a direct orbit average. It is found that the numerical accuracy of the new method is significantly higher than that of the direct orbit average method and overcomes the stiff numerical difficulties near trapped-passing boundaries.

Since the extracted NE is already a function of CoMs. A direct transformation from f_{NE} to a function of CoM can be done by a simple interpolation along a certain poloidal angle θ, which is shown in Figure 2.

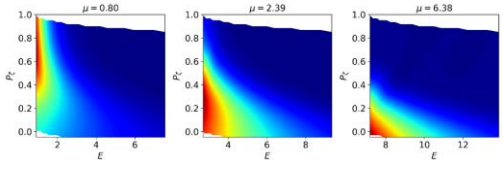

Figure 2: transformation to CoM space

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