9th Asia-Pacific Conference on Plasma Physics, 21-26 Sep, 2025 at Fukuoka

Correction of Numerical Errors at Current Sources in Explicit Finite-Difference Time-Domain Method for Plasma Kinetic Simulations

Harune Sekido[1], Takayuki Umeda[2]
[1] Institute for Space-Earth Environmental Research, Nagoya University, [2] Information Initiative Center, Hokkaido University
e-mail (speaker): sekido.harune.m9@a.mail.nagoya-u.ac.jp

The Finite-Difference Time-Domain (FDTD) method[1,2] is a numerical method for solving the time development of electromagnetic fields by approximating Maxwell's equations in both time and space using the second-order accurate finite differences. A staggered grid system is used in the FDTD method, in which Gauss's law is always satisfied. Owing to this advantage, the FDTD method is used for over half a century and applied to plasma kinetic simulations.

In the FDTD method, however, numerical oscillations occur due to the error between the numerical phase velocity and the theoretical phase velocity. The FDTD(2,4) method, which uses the fourth-order spatial differences, is proposed for reduction of the numerical errors.[3,4] However, the Courant condition becomes more restricted with higher-order finite differences in space and a larger number of dimensions.

Recently, advanced numerical methods have been developed by adding odd-degree difference terms to the time-development equations of FDTD, which relax the Courant condition and reduce numerical errors in phase velocity.[5,6,7] With those methods, computational time is reduced significantly, since longer time steps can be used. However, it has been found that large numerical errors arise in the charge conservation law if the time-development equations including current sources are discretized with higher-order finite differences in space. The numerical errors negatively affect plasma kinetic simulations. Therefore, corrections of the input current densities is necessary. In the present study, such numerical errors are suppressed by adding correction terms to the time-development equations of the FDTD methods with higher-order differences.

Figure 1 shows the results of numerical simulations in two dimensions. A loop current density is introduced at the center of the simulation domain. Panels (a) and (b) show the spatial profiles of the magnetic field component B_z using Sekido24 at $C = 1$ without correction terms for current densities and with correction terms, respectively. Here, Sekido24 is the FDTD scheme which has been proposed in the previous study.[7] Panel (a) shows that numerical errors appear at the position of current sources. Panel (b) shows that the numerical errors do not appear at current sources. The numerical errors occur for the case with $\nabla \cdot \boldsymbol{J} \neq \mathbf{0}$ because of higher-order finite differences in space. These errors can be suppressed with correction terms for current densities.

References

[1] K. S. Yee, IEEE Transactions on Antennas and Propagation, 14, 3, 302–307 (1966)
[2] A. Taflove, IEEE Transactions on Electromagnetic Compatibility, 22, 3, 191–202 (1980)
[3] J. Fang, Ph.D. dissertation, "Time domain finite difference computations for Maxwell's equations" (1989)
[4] P. G. Petropoulos, IEEE Transactions on Antennas and Propagation, 42, 6, 859–862 (1994)
[5] H. Sekido and T. Umeda, IEEE Transactions on Antennas and Propagation, 71, 2, 1630–1639 (2023)
[6] H. Sekido and T. Umeda, Progress In Electromagnetics Research M, 123, 83–93 (2024)
[7] H. Sekido and T. Umeda, Earth, Planets and Space, 76, 5 (2024)

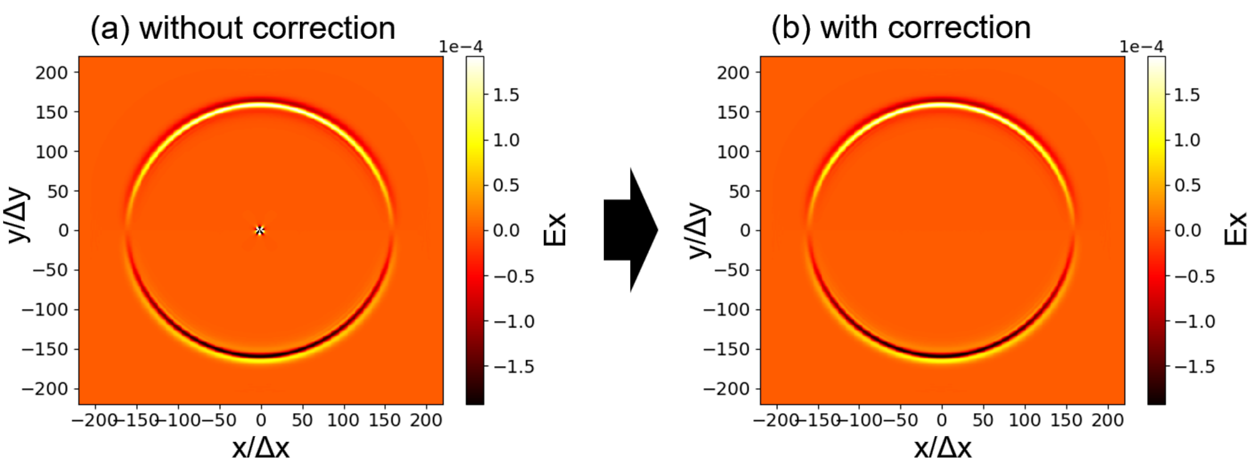

Figure 1. Spatial profiles of electric field E_x at $t = 200\Delta t/C$ with Sekido24 at $C = 1$ in two dimensions: (a) without correction terms for current densities; (b) with correction terms.