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Stationary Power-law Solutions of Weak Kinetic-Alfvénic Turbulence

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Plasma turbulence characterized by low-frequency Alfvénic oscillations is pervasive in space, astrophysical, and laboratory plasma environments as miscellaneous as the solar wind, the solar corona, interstellar media, and fusion devices. Among them, the solar wind has been found to be highly turbulent with Kolmogorov-like power-law spectra spanning a vast range of spatial scales. The turbulent cascade at kinetic scales of the solar wind attributed to kinetic Alfvén waves (KAWs) has brought up issues on its dispersive properties of the characteristic wave, the multiscale spectral features, the intermittent structure formation, and damping mechanisms.

Recent observations by Parker Solar Probe spacecraft[1] have suggested that, the plasma heating at the ion-gyroscale and the formation of a spectral transition region for a strongly imbalanced turbulence may have connections to the nontrivial cascade dynamics of KAWs and the conservation of helicity. At kinetic scales the helicity is subject to an inverse cascade, preventing the KAW turbulence from cascading to sub-ion scales. On the other side, previous analytical works[2] regarding the resonant parametric decay process of three-KAWs suggest that counter-propagating KAWs predominantly cascade to smaller perpendicular wavenumber region, and co-propagating KAWs exhibit a dual-character decay in the perpendicular wavenumber space, implying the inverse cascade of the helicity invariant in a multiple-mode system. To this end we discuss a potential substitute mechanism for the existence of the transition zone due to the nonlinear interactions among co-propagating KAWs. This idea has been elaborated in the strong turbulence regime[3] and remains to be explored in the weak turbulence regime.

Based on a nonlinear mode equation[2] derived from gyrokinetics to describe the turbulent evolution of KAWs in a low-β uniform plasma, we construct the weak turbulence description of KAW turbulence adopting the canonical Hamiltonian wave formulation. The system conserves the turbulent energy and the parallel-momentum (helicity). A wave kinetic equation (WKE) for the wave-action density $n_k = E_k/\omega_k$ is obtained within the random phase approximation (RPA). For the cases of counter-propagating (balanced) turbulence and all waves co-propagating, the WKE can be reduced into several limiting forms such as the long-wavelength-limit $k_\perp \rho_i \ll 1$ and the short-wavelength-limit $k_\perp \rho_i \gg 1$ where the system is scale-invariant or nearly scale-invariant. Then we can calculate the Kolmogorov-Zakharov (KZ) spectra[4] corresponding to energy and parallel-momentum cascades by applying the conformal Kuznetsov-Zakharov transformation for axisymmetric anisotropic systems. For the balanced turbulence, the energy cascade gives two stationary spectrum $n_k \propto k_z^{-3/2} k_\perp^{-3}$ and $n_k \propto k_z^{-3/2} k_\perp^{-9/2}$ in the $k_\perp \rho_i \ll 1$ and $k_\perp \rho_i \gg 1$ limits, respectively. For the case of all KAWs co-propagating, the KZ solutions corresponding to energy cascade and parallel-momentum cascade in the $k_\perp \rho_i \gg 1$ limit are $n_k \propto k_z^{-3/2} k_\perp^{-9/2}$ and $n_k \propto k_z^{-3/2} k_\perp^{-4}$ respectively, and the energy-cascade spectrum and momentum-cascade spectrum in the $k_\perp \rho_i \ll 1$ limit are obtained as $n_k \propto k_z^{-3/2} k_\perp^{-5}$ and $n_k \propto k_z^{-3/2} k_\perp^{-4}$. It is found that the energy is always transferred towards small scales and in the co-propagating situation the parallel-momentum exhibits an inverse cascade. We further discuss the validity of weak turbulence assumption and numerically calculate the WKE to verify the existences and cascade directions of the KZ solutions[5].

The KZ spectra in different limits of the model illustrate the multiscale character of weak KAW turbulence across the ion-gyroscale. The possible existence of inverse momentum-cascade spectra for the co-propagating case sheds some light on the potential explanation about the transition region spectrum of the solar wind turbulence. The weak turbulence theory and KZ spectra serve as a strict mathematical tool on analyzing stationary spectra, cascade directions, locality criteria and the validity of KAW turbulence. The inverse cascade process might further result in the development of self-organization states of various quasi-two-dimensional turbulent systems in space and galaxies, as well as the effective energy transfer and plasma heating in solar physics and magnetically confined systems.

References

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