



## **Large-scale multifractality and non-self-similar energy decay in one-dimensional (1D) Burgers and three-dimensional (3D) Navier-Stokes turbulence**

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We present studies of freely decaying turbulence in the one-dimensional (1D) Burgers equation (henceforth, Burgulence) and the three-dimensional (3D) Navier-Stokes (3D NS) equation (henceforth NS turbulence). We first investigate the decay in time  $t$  of the energy  $E(t)$  in Burgulence, for a fractional Brownian initial potential, with Hurst exponent  $H$ ; here, we provide a rigorous proof of the self-similar time-decay of  $E(t)$ , which had been determined only heuristically so far. We show that this self-similar decay is a consequence of the nontrivial boundedness of the energy for any positive time. We next define a spatially forgetful oblivious fractional Brownian motion (OFBM), with Hurst exponent  $H$ , and prove that Burgulence, with an OFBM as initial potential  $\varphi_0(x)$ , is not only intermittent, but it also displays, a hitherto unanticipated, large-scale bifractality or multifractality; the latter occurs if we combine OFBMs, with different values of  $H$ . This is the *first rigorous proof of genuine multifractality for turbulence in a nonlinear hydrodynamical partial differential equation*. We then present direct numerical simulations (DNSs) of freely decaying turbulence, which capture some aspects of such multifractality. For Burgulence, we investigate the decay of  $E(t)$  for two cases: (A)  $\varphi_0(x)$  a multifractal random walk [a generalization of that crosses over to a fractional Brownian motion beyond a crossover scale  $L$ , which we can tune to go from small- to large-scale multifractality; (B) initial energy spectra  $E_0(k)$ , with wavenumber  $k$ , having one or more power-law regions, which lead, respectively, to self-similar and non-self-similar energy decay. Our analogous DNSs of the 3D NS equations also uncover self-similar and non-self-similar energy decay. Challenges confronting the detection of genuine large-scale multifractality, in numerical and experimental studies of 3D NS and magnetohydrodynamic (MHD) turbulence, are highlighted.

Navier-Stokes turbulence, Takeshi Matsumoto, Dipankar Roy, Konstantin Khanin, Rahul Pandit, Uriel Frisch, <https://doi.org/10.48550/arXiv.2503.08983>.

### References

- [1] Large-scale multifractality and lack of self-similar decay for Burgers and 3D