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MHD analysis of electromagnetic GAMs in up-down asymmetric tokamaks

Zhe Chen[1], Yixiang Li[1], Haijun Ren[1,*] and Hao Wang[2]
[1] University of Science and Technology of China
[2] National Institute for Fusion Science
e-mail (speaker): hjren@ustc.edu.cn

Geodesic acoustic modes (GAMs), high-frequency zonal flows in toroidal fusion plasmas, are essential for turbulence regulation and transport control[1]. In realistic tokamaks, non-circular flux surfaces arising from elongation, triangularity, and asymmetry significantly modify GAM frequencies and perturbations. Yet, analytical studies of electromagnetic GAMs[2] in such geometries remain scarce. Here, we perform an MHD analysis of electromagnetic GAMs in up-down asymmetric, non-circular tokamaks using a Miller-like equilibrium model.[3] In the long-wavelength and local limits, the GAM governing equations are derived from the ideal MHD framework as follows:

$$\left(1+\frac{\gamma p}{B^2}\right)\phi+\frac{c_s^2}{\omega^2}B\nabla_\parallel\left(B^{-1}\nabla_\parallel\phi\right)+2\mathcal{K}_\theta g^{11}\xi_\theta=0,$$
$$\xi_\theta+\frac{1}{\rho\omega^2}\frac{B^2}{g^{11}}\nabla_\parallel\left[B^{-1}g^{11}\nabla_\parallel(B\xi_\theta)\right]+2\frac{c_s^2}{\omega^2}\mathcal{K}_\theta\phi=0. \quad (1)$$

Here, $\gamma, p, \rho, B, c_s=\sqrt{\gamma p/\rho}, \omega, \vec{\mathcal{K}}$ and $\vec{\xi}$ are adiabatic index, pressure, density, magnetic field, sound speed, mode frequency, magnetic curvature and Lagrangian displacement, respectively. $\phi=\nabla\cdot\vec{\xi}$, $g^{11}=\nabla r\cdot\nabla r$ and the subscript θ indicates the poloidal component.

The Miller-like flux surface is described as follows[4]:

$$R=R_0(r)+r\cos\left[\theta+\arcsin\delta(r)\sin\theta\right]+\sigma(r)r\sin\theta,$$
$$Z=\kappa(r)r\sin\theta. \quad (2)$$

Here, κ, δ and σ are the elongation, triangularity and asymmetry parameter. Because $R(r,\theta)\neq R(r,-\theta)$ for $\sigma\neq 0$, σ represents the up-down asymmetry of the cross-section. Assuming weakly noncircular flux surface and low β_0 (thermal to magnetic pressure ratio), explicit analytical forms for the $\vec{\xi}$, ω and magnetic perturbations are obtained.[3] Among them, the poloidal magnetic perturbation $\hat{B}_\theta$ is frequently observed in diagnostics of GAM, and its expression is derived as:

$$\frac{\hat{B}_\theta}{B_0}=-q\beta_0\frac{\xi_{\theta,0}}{R_0}\left[\frac{9\varepsilon-5\delta-s_\delta}{2}\sin\theta+\sin 2\theta+\frac{4\Delta'+2\delta-\varepsilon-s_\delta}{6}\sin 3\theta\right.$$
$$-\frac{4\eta+3s_\kappa}{8}\sin 4\theta+\frac{3\delta+2s_\delta}{10}\sin 5\theta-\frac{s_\sigma}{4}-\sigma\cos 2\theta \quad (3)$$
$$\left.-\frac{4\sigma+3s_\sigma}{8}\cos 4\theta\right]+\mathcal{O}\left(\frac{\beta_0^2 q\xi_{\theta,0}}{R_0}\right)+\mathcal{O}\left(\frac{\lambda^2\beta_0 q\xi_{\theta,0}}{R_0}\right).$$

Here, q is safety factor, $\eta=\kappa-1$, $\xi_{\theta,0}$ is the θ-independent part of $\xi_\theta R_0/R$. ε and Δ' are the inverse aspect ratio and Shafranov shift gradient. s_κ, s_δ and s_σ represent radial gradients of κ, δ and σ. All shaping parameters $(\varepsilon,\eta,\delta,\Delta',\sigma,s_\kappa,s_\delta,s_\sigma)$ are assumed to be of order $O(\lambda)$ for weakly noncircular flux surface, with λ being a small expansion parameter for asymptotic analysis.

Equation (3) indicates that $\hat{B}_\theta$ is primarily dominated by the $\sin 2\theta$ term, independent of shaping effects. Up-down asymmetry (σ) introduces $\cos 2\theta$ and $\cos 4\theta$ components, making $\hat{B}_\theta$ asymmetric when $\sigma\neq 0$. The amplitude ratio of $\cos 2\theta$ to $\sin 2\theta$ components is σ and the phase shift is π, consistent with MHD simulations[4]. Other shaping parameters induce $\sin m\theta$ components $(m=1,3,4,5)$ [2], among which the $\sin\theta$ component is most significant due to large coefficient $(9\varepsilon-5\delta-s_\delta)/2$. For large ε or negative δ, the $\sin\theta$ term can rival or exceed the $\sin 2\theta$ component, as shown in Figure 1(a). Kinetic simulations[5] yield an amplitude ratio (between $\sin\theta$ and $\sin 2\theta$ components) of 0.59, closely matching the analytical result of 0.61, as seen in Figure 1(b).

References
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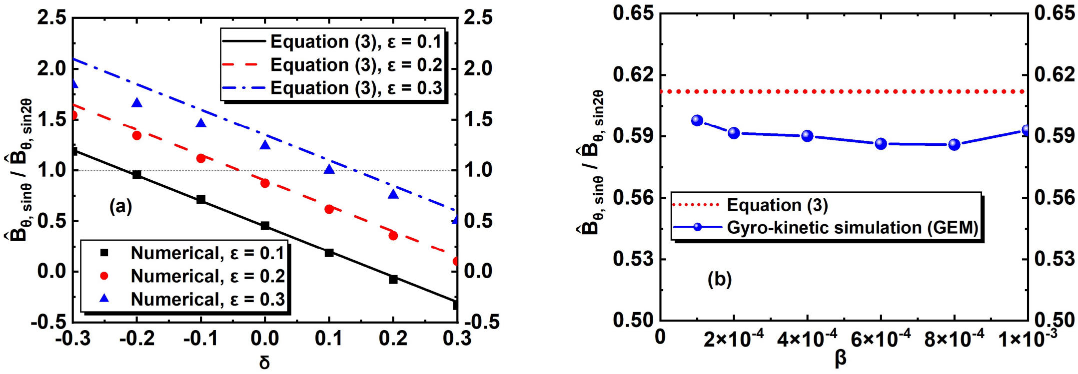

Figure 1. Ratio of the $\sin\theta$ to the $\sin 2\theta$ components of $\hat{B}_\theta$. (a) Ratio versus δ. Parameters: $q=4, \kappa=1, \Delta'=s_\kappa=s_\delta=\sigma=s_\sigma=0$. Numerical spots are solutions of equation (1) without assuming the small parameter λ. (b) Ratio versus $\beta(\equiv 2p/B_0^2)$ $for\ q=4, \varepsilon=0.136$ and circular flux surfaces. Simulation results are from Figure 18 of Ref.[5].