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Suppressing phase velocity errors in higher-order Finite-Difference Time-Domain methods

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The Finite-Difference Time-Domain (FDTD) method is a fundamental approach in numerical simulations of electromagnetic waves. The method has advantages in its simplicity of the algorithm and its suitability for time-domain analysis. Moreover, by using a staggered Yee grid in the spatial difference, it inherently satisfies Gauss's law for both electric and magnetic fields [1]. To reduce numerical oscillations, FDTD schemes with fourth-order spatial difference have been developed [2]. However, high-order methods often require stricter Courant–Friedrichs–Lewy (CFL) conditions and may arise numerical errors in current density. To enable more accurate and efficient plasma simulations with less computational costs, it is important to relax CFL conditions in a higher-order FDTD methods.

Previous study has demonstrated that third-degree difference operators considering Laplacian can effectively relax CFL conditions and reduce anisotropic phase velocity errors in the fourth-order FDTD [2]. To further suppress phase velocity errors and thereby reduce numerical oscillations in implementations, the same approach was applied to a sixth-order FDTD scheme. Although the CFL conditions can be relaxed, the expected reduction in phase velocity error was not achieved. A subsequent dispersion relation analysis revealed that the sixth-order scheme is unable to simultaneously minimize phase velocity errors at both $(k_x, k_y) = (\pi, \pi)$ and $(\pi, 0)$ in the wavenumber domain. In contrast, the fourth-order scheme naturally satisfies both conditions.

In this study, to satisfy conditions that minimize phase velocity errors at $(k_x, k_y) = (\pi, \pi)$ and $(\pi, 0)$, we introduce additional degrees of freedom into the 6th-order scheme by adding operators of second- and fourth- order along with third-order considering Laplacian. Although numerical results have not yet been obtained, preliminary dispersion relation analysis suggests this method may lead to improvements in both the numerical stability and reduction in phase velocity error of current high-order FDTD schemes.

This study focuses on reducing numerical errors in high-order FDTD methods while relaxing the CFL conditions. I hope that this presentation will help to explore additional analytical techniques and future developments.

References

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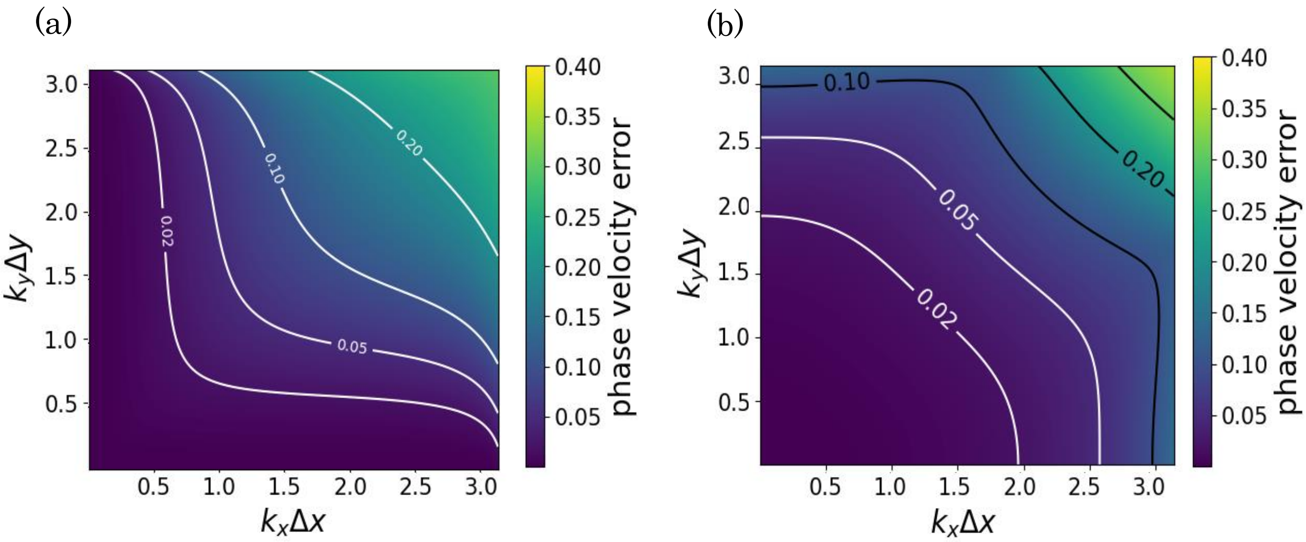

Fig. Dependence of the phase velocity errors on wavenumber in two dimensions with Courant number = 1: (a) FDTD 4th-order in spatial difference + Laplacian operator; (b) FDTD 6th-order in spatial difference + Laplacian operator