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Linear Theory of Tearing Instability with the improved WKB approximation

Tohru Shimizu[1]

[1] RCSCE of Ehime University, Japan

shimizu.toru.mu@ehime-u.ac.jp

Tearing Instability is a resistive-MHD (Magneto Hydro Dynamic) instability of the current sheet often observed in space plasmas and experimental plasmas. It is believed that plasma explosions, such as substorms and solar flares, are triggered by the instability. However, the triggering condition is still unclear. To specify the condition, the MHD linear theory of the tearing instability is studied during the past 60 years. Using WKB (Wentzel Kramers Brillouin) approximation, Loureiro largely improved the previous studies [1]. Then, the perturbation equation derived by Loureiro was numerically studied by Shimizu [2]. Then, extending the Loureiro's work, he found the critical (i.e., triggering) condition [2]. This paper shows another extension of the Shimizu's works, where the WKB approximation introduced by Loureiro is improved to explore the low κ range, i.e., $\kappa \sim 0$. At this point, the WKB studied in [2] fails in $\kappa \sim 0$.

The perturbation equations studied in this paper are shown as follows.

$$N\Phi'''' = \kappa\varepsilon^2((\lambda + 2\kappa N)\Phi'' - (\lambda + \kappa N)\kappa^2\varepsilon^2\Phi + f(\xi)(\Psi'' - \kappa^2\varepsilon^2\Psi) - \xi\Phi'''/\kappa + \kappa\varepsilon^2\xi\Phi' + 2\kappa\varepsilon^2\Phi - f''(\xi)\Psi) \quad (1)$$

$$\Psi'' - \kappa^2\varepsilon^2\Psi = \kappa\lambda\Psi - \xi\Psi' - \kappa f(\xi)\Phi \quad (2)$$

Excepting the viscosity N term, these equations are essentially Eqs.(6) and (7) in [1] but some terms have been translated following the notations of Eqs.(8) and (9) in [1]. For the details of the notations of Eqs.(1) and (2), refer to [2].

Loureiro did not directly solve Eqs.(6) and (7) in [1] and hence Eqs.(1) and (2). Instead, he solved Eqs.(8) and (9) which is for the lowest-order WKB approx.. Also, Shimizu could not solve them in [2] and [3]. However, <u>the viscosity term introduced by Shimizu [2] makes Eqs.(1) and (2) solvable</u>. In fact, Eqs.(1) and (2) can be numerically solved to find Φ and Ψ, when a set of $\Phi(0)$, $\Phi'(0)$, $\Phi''(0)$, $\Phi'''(0)$, $\Psi(0)$, and $\Psi'(0)$ are given. How to find the zero- contact solutions of Φ and Ψ is explained in [2].

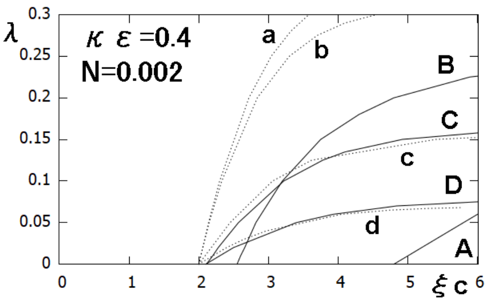

Figure 1: The change of linear growth rate λ, keeping $\kappa\varepsilon = 0.4$, and N=0.002.

Figure 1 shows how λ changes with respect to <u>the location ξc of the upstream open boundary [2]</u>. The figure format is the same as Fig.8 in [2]. Lines "a-d" are for the lowest-order of WKB and correspond to Case3 of [2]. Lines "A-B" are obtained for Eqs.(1) and (2). Lines "a" and "A" are for $\kappa = 2$, Lines "b" and "B" are for 4, Lines "c" and "C" are for 16, and Lines "d" and "D" are for 40. Hence, Figure 1 shows that the improvement of WKB approx. effectively stabilizes the current sheet in low κ range, because λ is reduced from lines "a,b" to "A,B". Inversely, since Lines "c,d" are respectively close to "C,D", the lowest-order (not-improved) WKB in [2] are not so bad in higher κ range, i.e. $\kappa > 10$.

Figure 2 shows the critical condition (i.e., $\lambda = 0$) of the tearing instability. In fact, the upper-left region of these three lines is stable and the lower-right is unstable. WKB0 is for Case 3 in [2], i.e., the lowest-order WKB, which is the same as Fig.16 in [2]. Then, WKB1a and 1b are respectively for $\varepsilon = 0.025$ and 0.1 in Eqs.(1) and (2). WKB1a and 1b are close but the slight difference is important. Because, Eqs.(1) and (2) separately depends on κ and ε even in $\lambda = 0$, WKB0 depends only on the value of $\kappa\varepsilon$.

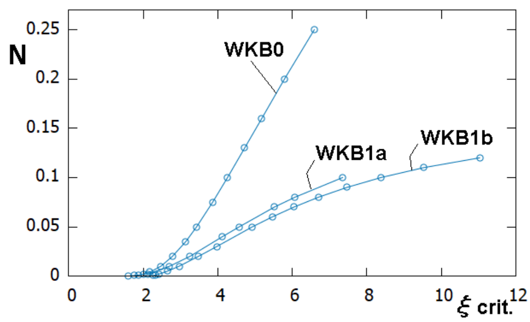

Figure 2: Viscosity N for the critical ξ **c**.

Summary:

Figures 1 and 2 are qualitatively similar to the results reported in [2] but show that the current sheet tends to be more stable in the improvement of the WKB approximation, i.e., from (15) and (16) in [2] to Eqs.(1) and (2). It suggests that <u>the tearing instability observed in fully non-linear MHD is less easily caused</u> than the prediction of the linear theory reported in [2], and [3].

References:

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