9th Asia-Pacific Conference on Plasma Physics, 21-26 Sep, 2025 at Fukuoka

Modeling Nonlinear and Chaotic Dynamics with Interpretable Data-Driven Reduced Order Models

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Understanding and modeling the dynamics of complex nonlinear systems, whether governed by partial or ordinary differential equations, remains a central challenge across scientific disciplines. Traditional reduced order models (ROMs), such as Proper Orthogonal Decomposition (POD) combined with Galerkin projection, effectively reduce system dimensionality but often struggle with stability and accurately capturing nonlinear behavior[1].

To address these limitations, we propose an interpretable machine learning framework based on Layered Polynomial Neural Networks (LPNNs), inspired by recent advances in deep polynomial neural networks[2] and interpretable Polynomial Neural ODEs (PolyNODEs)[3]. LPNNs are designed to model multivariate polynomial systems with scalability and robustness. The architecture employs a hierarchical structure of sequential shallow neural networks, each responsible for modeling polynomial terms of increasing degree. This layered design enhances expressivity while maintaining interpretability and dynamically adjusts the number of parameters to match system complexity, effectively mitigating underparameterization challenges.

The effectiveness of LPNNs is demonstrated through two benchmark problems: the Burgers' equation[4], a nonlinear PDE modeling advection-diffusion phenomena, and the Lorenz system[5], a canonical chaotic ODE system. In the Burgers' equation case, after applying POD to extract dominant modes, LPNNs learn the evolution of reduced-order coefficients, effectively capturing both large-scale advection and small-scale diffusion dynamics with a low mean squared error (MSE). Essentially, LPNNs learns the closure of the Galerkin ROMs to accurately reconstruct the nonlinear dynamics without explicit knowledge of the governing equations.

For the Lorenz system, LPNNs successfully capture the chaotic behavior (see Figure 1), accurately reconstructing the governing equations and reproducing the butterfly-shaped attractor, thus modeling the sensitive dependence on initial conditions inherent to chaos.

Despite the strong performance, scalability challenges remain due to the combinatorial growth of polynomial terms in high-dimensional systems, leading to significant computational demands. Future work will focus on integrating additional sparsity techniques and extending the framework to even more complex nonlinear and high-dimensional dynamical systems.

References

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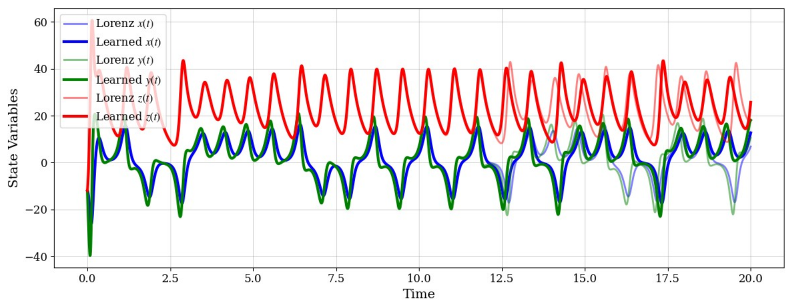

Figure 1. Trajectories of the Lorenz system (transparent) and the learned model (solid) for *x(t)* (blue), *y(t)* (green), and *z(t)* (red), showing the model's ability to capture the system's dynamics.