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A rotation-free vortex solution in special and general relativistic hydrodynamics

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We present an exact vortex solution in relativistic hydrodynamic flows. This solution, which possesses a vacuum core at the center, corresponds to a point vortex solution in non-relativistic flows. A schematic figure of the core structure of the vortex solution is provided in Figure 1. The flow just outside of the vacuum core is supersonic, the flow outside of that is transonic, and the flow furthest from the core is subsonic. A similar structure is also found in non-relativistic compressible point vortex flows [1]; however, the core radius, which is a constant in non-relativistic flows, depends on the sonic speed of the system in relativistic flows and becomes larger for a fixed polytropic exponent [2].

We also calculate various physical quantities such as density, pressure, and sound speed as functions of the distance from the center of the core and compare them with those in non-relativistic flows.

We can construct a similar vortex solution in massless radiation fluids, in which we show that the core radius in radiation fluids is proportional to the vortex strength.

As an application of the vacuum-core vortex in relativistic flows, we consider a mathematical model including gravity [3,4]. We show that this model provides a solution to describe a rotation-free torus corresponding to an accretion disk around a black hole (Figure 2).

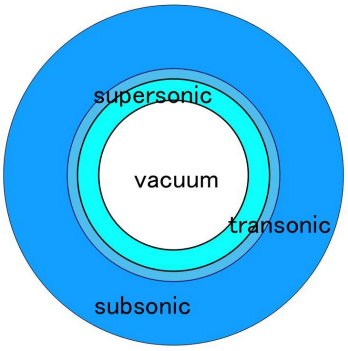

Figure 1.

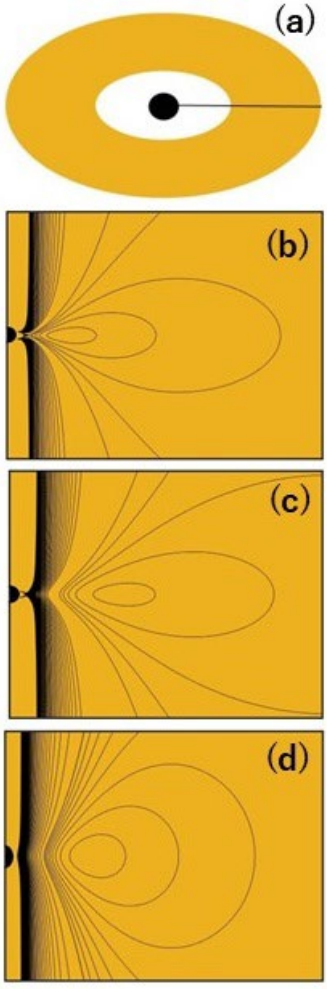

Figure 2.

References

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Figure 1. Schematic figure of a core structure in compressible potential flows, where regions from high speed to low speed (from supersonic to subsonic) spread in order outside of the core.

Figure 2. (a) Schematic figure of a black hole and an accretion disk. The central black hole (black circle) is surrounded by a distorted torus-shaped accretion disk. (b) – (d) denote various (poloidal) cross sections of (a) cut along the solid line. The curves represent iso-enthalpy surfaces. The vortex strength increases in the order from (b) to (d).