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Kinetic full wave analysis in inhomogeneous plasmas using integral form of dielectric tensor

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Frequency-domain full wave analyses in confined plasmas have been widely used for systematic and efficient description of the wave behavior including tunneling of an evanescent layer, formation of standing waves and coupling to a finite-size antenna. In order to describe various wave-particle interactions, kinetic dielectric tensor has to be employed.

Conventional kinetic dielectric tensor has been derived, however, in a uniform plasma and expressed as a function of wave number. Since the wave number is unknown a priori, several schemes, such as the use of approximate wave number in cold plasmas, replacing the wave number by differential operator, and Fourier mode expansion, have been proposed and employed. Since the previous schemes are based on the kinetic dielectric tensor in a uniform plasma, it is difficult to describe the wave behavior near the cyclotron resonance in an inhomogeneous plasma, and waves with short wave-length, such as Bernstein waves.

In order to describe the kinetic response in an inhomogeneous plasma, we have systematically formulated integral form of dielectric tensor. From the unperturbed orbit in an inhomogeneous plasma, the particle velocity $\boldsymbol{v}$ is expressed by the position $\boldsymbol{r}'$ at time t'. By replacing the velocity integral by the positional integral over $\boldsymbol{r}'$, we obtain the induced current $\boldsymbol{J}(\boldsymbol{r})$ expressed by the integral of the conductivity $\overleftrightarrow{\sigma}(\boldsymbol{r},\boldsymbol{r}')$ and the wave electric field $\boldsymbol{E}(\boldsymbol{r}')$.

$$\boldsymbol{J}(\boldsymbol{r}) = \int \mathrm{d}\boldsymbol{r}' \, \overleftrightarrow{\sigma}(\boldsymbol{r},\boldsymbol{r}') \cdot \boldsymbol{E}(\boldsymbol{r}')$$

For parallel motion in a magnetized plasma, the integral form of the dielectric tensor for Maxwellian plasmas can be expressed with the plasma dispersion kernel function (PDKF)

$$U_n(\xi,\eta,\nu_m) = \frac{\mathrm{i}}{\sqrt{2\pi}} \int_0^\infty \mathrm{d}\tau \, \tau^{n-1} \exp\left[-\frac{1}{2}\frac{\xi^2}{\tau^2} - \frac{1}{2}\eta^2\tau^2 + \mathrm{i}\,\nu_m\tau\right]$$

where $\xi = (z-z')/\omega v_{\mathrm{T}}$, η depends on the inhomogeneity scale length, the wave number in the uniform direction, the difference between the wave frequency and the cyclotron harmonics $n\omega_{\mathrm{c}}$, $\nu = (\omega - m\omega_{\mathrm{c}})/\omega$ and the thermal velocity v_{T}. This kernel function is related to the inverse Fourier transform the plasma dispersion function.

For perpendicular motion in a magnetized plasma, the dielectric tensor can be expressed with the plasma gyro kernel functions (PGKF)

$$F_n^{(i)}(X,Y) = \frac{1}{2\pi^2}\int_0^\pi \mathrm{d}\theta_g \left[-\frac{X^2}{1+\cos\theta_g} - \frac{Y^2}{1-\cos\theta_g}\right] f_n^{(i)}(\theta_g)$$

where $f_n^{(i)} = [\cos n\theta_g/\sin\theta_g, \sin n\theta_g, \sin n\theta_g/\sin^2\theta_g, \cos\theta_g \sin n\theta_g/\sin^2\theta_g]$, $X = [(x+x')/2 - x_0]\omega_c/v_{\mathrm{T}}$, $\xi = (z-z')/\omega v_{\mathrm{T}}$, and x_0 is the position of guiding center. This kernel function is related to the inverse Fourier transport of the modified Bessel function.

The example of the use of PDKF is the description of magnetic beach heating near the electron cyclotron resonance (ECR) shown Fig.1. The magnetic field strength is linearly increasing in z. The right-hand circularly polarized wave approaching from the high field side is absorbed near the ECR.

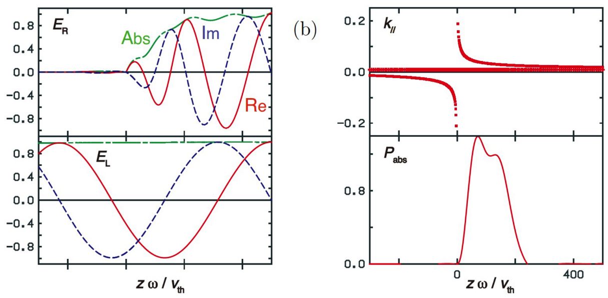

Fig.1: R and L circularly polarized components, parallel wave number and power deposition profile

The example of the used of PGKF is the analysis of O-X-B mode conversion of EC waves in tokamak configuration shown in Fig.2. When the O-mode with optimum injection angle is excited from the low-field-side, it is converted to the X-mode, reflected in the high field region, reflected again near the upper hybrid layer, converted to the Bernstein wave, and finally absorbed near the cyclotron resonance near.

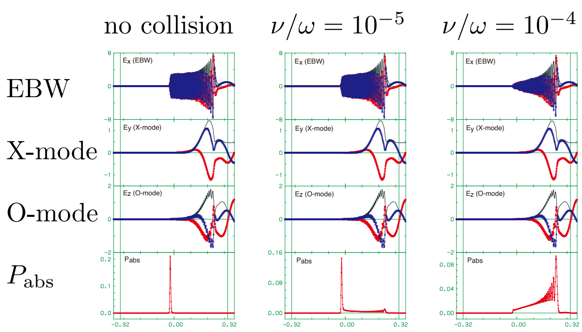

Fig.2: Wave electric field and power deposition profile of O-X-B mode conversion