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The metriplectic 4-bracket and the unified thermodynamic (UT) algorithm: applications and computations

Philip J. Morrison
Department of Physics and IFS, The University of Texas at Austin, Austin, TX
e-mail (speaker): morrison@physics.utexas.edu

Metriplectic dynamics, introduced in [1], provides a geometric framework for thermodynamically consistent (TC) dynamical systems. Such systems conserve an energy (Hamiltonian) functional and have an entropy functional that monotonically increases in time. Thus, the dynamics of TC systems satisfies the first and second laws of thermodynamics.

Constructing or deriving macroscopic TC models can be challenging, particularly for complex fluid and plasma systems. Metriplectic dynamics can serve as a useful guide and has been used to include a variety of thermodynamic processes in extended MHD [2].

Recently, a significant advance was made that allows the construction of TC systems in a more direct manner. This was done by introducing the metriplectic 4-bracket [3], which provides a geometric structure that combines properties of Poisson manifolds (dissipationless phase spaces) with degenerate Riemannian manifolds. In this setting the rate of entropy production is given by the sectional curvature defined by the Hamiltonian and Entropy level sets.

A unified thermodynamic (UT) algorithm [4], based on the metriplectic 4-bracket, has been obtained which is of practical use. We have used it in variety of applications, ranging from complex multiphase fluid flow to collision operators in guiding center phase space [4]-[7]. In all cases we have either reproduced, corrected, or extended results in the literature.

The metriplectic framework also provides a convenient way to devise numerical algorithms based on structure. In [8] it has been used to obtain a variety of equilibrium states by a relaxation method.

The metriplectic 4-bracket provides a particularly convenient path for devising numerical discretizations that maintain TC. Rather than the usual conservative (finite volume) scheme for shock capturing, the 4-bracket can produce an entropy producing and energy conserving scheme directly [9].

We will discuss recent mathematical results, analytical and geometric, and how to implement the UT algorithm in a variety of contexts. In addition, we will discuss examples of 4-bracket numerical implementation.

References

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