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Canonical Hamiltonian Theory and Symplectic Algorithms of Guiding Center Dynamics

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The guiding-center model is highly effective in addressing the multi-scale dynamics of magnetized plasmas and has been widely applied in magnetic confinement fusion research and other magnetized plasma problems. However, for a long time, the universal canonical formation and canonical Hamiltonian structure of guiding-center dynamics could not be effectively constructed. In recent work, we have rigorously demonstrated that guiding-center dynamics can be generally expressed as a canonical Hamiltonian system with two constraints in a 6-dimensional phase space, and the solution flow of the guiding-center system lies on a canonical symplectic submanifold. Therefore, the guiding center can be regarded as a pseudo-particle with an intrinsic magnetic moment. This model can correctly and independently describe the dynamics of plasma systems on time scales larger than the gyro-period, fully determining the dynamical behavior of the guiding-center system and clearly deriving the velocity and force of the guiding-center pseudo-particle. Based on this theory, a series of related theories such as symplectic algorithms for guiding centers, canonical gyrokinetic theory, and canonical PIC algorithms can be systematically developed. This canonical guiding-center theory also provides insights into the origin of the intrinsic magnetic moment of particles. Based on the general symplectic Hamiltonian theory, we develop symplectic Runge-Kutta methods for guiding center dynamics. Theoretical analysis confirms that when Runge-Kutta methods satisfying specific conditions are applied to guiding center dynamics, they can precisely conserve the symplectic structure.

References

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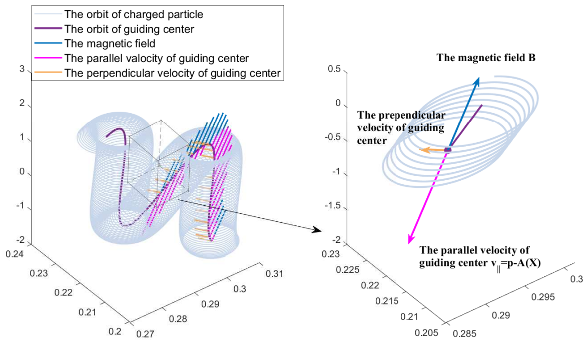

Fig.1 The orbits of a guiding center pseudo-particle (in purple) in an external magnetic field, with corresponding charged particle (in grey) for reference. The vectors of magnetic field (in blue), p − A(X) (in pink), and perpendicular velocity of the guiding center (in orange) are plotted at different positions along the trajectory. A section of the orbits is zoomed in and shown in the right panel.

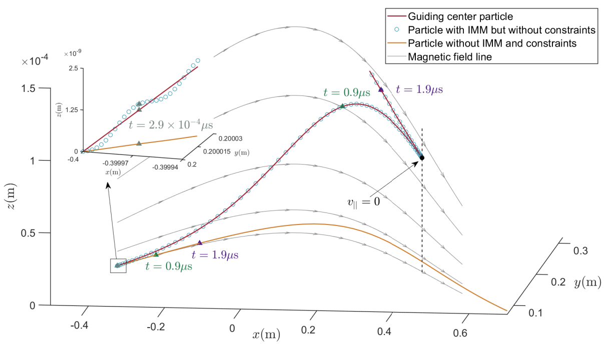

Fig.2 The trajectories of three different systems in a magnetic mirror field for comparison. The red curve depicts the trajectory of the canonical guiding center system. The orange curve depicts the trajectory of the system without IMM or constraints. And the light blue circles trace the orbit of the system with the complete Hamiltonian but without constraints. The triangles with different colors expose the spots of the three systems at different times, respectively. The zoomed-in window exhibits details of the fast timescale dynamics of the system with IMM but without constraints.

Note: Abstract should be in (full) double-columned one page.