

## Novel effects of kinetic and cross helicities in solar- and astro-physics

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Turbulent fluxes in (magneto-)hydrodynamic flows are analysed based on a closure theory of strongly non-linear, inhomogeneous and anisotropic turbulence. Turbulent helicities represent the geometrical and topological information of fluctuations, while the turbulent energies do the intensity one. Novel turbulent dynamo and momentum transport mechanisms relevant to circumstances ubiquitous in solar- and astro-physics will be argued.

Firstly, the effect of the local density of kinetic helicity  $H = \langle \mathbf{u}' \cdot \boldsymbol{\omega}' \rangle$  ( $\mathbf{u}'$ : velocity fluctuation,  $\boldsymbol{\omega}'$ : vorticity fluctuation,  $\langle \dots \rangle$ : ensemble average) in the stellar angular-momentum transport is discussed. In addition to the  $\alpha$  dynamo effect, the kinetic helicity, if coupled with the large-scale vortical motion  $\boldsymbol{\Omega}$  ( $= \nabla \times \mathbf{U}$ ) ( $\mathbf{U}$ : mean velocity) or rotation  $\boldsymbol{\omega}_F$ , is expected to contribute to the linear- and angular-momentum transport. The Reynolds and turbulent Maxwell stresses  $\mathbf{R} \equiv \langle \mathbf{u}' \mathbf{u}' \rangle - \langle \mathbf{b}' \mathbf{b}' \rangle$  are expressed as

$$\mathbf{R}_D = -\nu_T \mathbf{S} + \nu_M \mathbf{M} + [\boldsymbol{\Gamma} \boldsymbol{\Omega}_* + (\boldsymbol{\Gamma} \boldsymbol{\Omega})^\dagger]_D, \quad (1)$$

where  $\mathbf{b}'$  is the magnetic fluctuation in the Alfvén-speed unit, the indices D and  $\dagger$  denote the deviatoric part and transpose of a tensor,  $\mathbf{S}$  is the mean velocity strain,  $\mathbf{M}$  is the mean magnetic-field strain,  $\boldsymbol{\Omega}_*$  is the mean absolute vorticity ( $\boldsymbol{\Omega}_* = \boldsymbol{\Omega}_* + 2\boldsymbol{\omega}_F$ ),  $\nu_T$  is the eddy viscosity,  $\nu_M$  is the magnetic counterpart, and  $\boldsymbol{\Gamma}$  is the helicity-related transport coefficient, whose analytical and model expressions are obtained from the fundamental equations. The role of kinetic helicity in the stellar angular-momentum transport will be discussed. The large-scale azimuthal vorticity associated with the meridional circulation,  $\boldsymbol{\Omega}_{MC}$  (left), is coupled with the inhomogeneities of turbulent helicity,  $\partial H / \partial r$  (centre) and  $(1/r) \partial H / \partial \theta$  (right), contribute to the azimuthal acceleration of the large-scale velocity (Fig. 1) [1].

Secondly, in the magnetohydrodynamics (MHD), the effects of the local density of cross helicity  $\langle \mathbf{u}' \cdot \mathbf{b}' \rangle$  in dynamo and flow generation will be discussed. In combination of the Coriolis force and the turbulent cross helicity, an electromotive force is induced in the

direction of  $\boldsymbol{\Omega}_*$ . On the other hand, combination of the fluctuating Lorentz force  $\mathbf{J} \times \mathbf{b}'$  and the turbulent cross helicity induces an large-scale flow in the direction of  $\nabla \times \mathbf{J}$  ( $\mathbf{J}$ : mean electric-current density). These effects may play an important role in stellar convection zone with counter-balancing the usual turbulent viscosity.

Finally, at a very high or low magnetic Prandtl number  $Pm = \nu / \eta$  ( $\nu$ : viscosity,  $\eta$ : resistivity), some additional contribution to the  $\alpha$  effect may arise due to the cross responses between the velocity and magnetic fluctuations. This cross-response effect has not been fully explored in the previous dynamo studies (Fig. 2). Relevance of this effect in astrophysical phenomena such as the solar convection zone [ $Pm = O(10^{-7}) - O(10^{-4})$ ] and galaxies [ $Pm = O(10^{11})$ ] will be discussed [2].

In addition, by considering the production mechanisms of the kinetic and cross helicities, an interesting consequence of the co-existence of these helicities will be argued.

### References

- [1] Yokoi, N. and Miesch, M. S. Geophys. Astrophys. Fluid Dyn. **119**, 1-29 (2025).
- [2] Yokoi, N. Rev. Mod. Plasma Phys. **7**, 33 (2023).

Fig. 1: Meridional circulation and helicity inhomogeneity. Meridional circulation (Left), Radial (Centre) and latitudinal (Right) distributions of turbulent helicity.

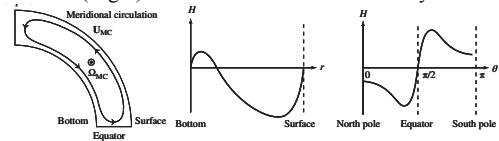


Fig. 2: Cross-interaction effect in the  $\alpha$  dynamo. Temporal evolutions of kinetic helicity and cross helicity (Upper), standard  $\alpha$  effect and cross-interaction  $\alpha$  effect (Lower).

