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Continuous Koopman Neural Operators for Drift-Reduced Braginskii Fluid Model

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The prediction and understanding of plasma dynamics across varying plasma regimes remains a fundamental challenge in fusion research [1]. While traditional approaches require expensive simulations for each parameter configuration, we present a novel application of continuous Koopman Neural Operators (cKNO) to learn the evolution operators parameterized by the collision parameter ν (which characterizes the plasma's collisional transport), enabling interpolation between different plasma behaviors without additional simulations.

We utilize 3D data generated from Braginskii model simulations [2], where the magnetic geometry and other plasma parameters are fixed, and only the collision parameter ν is varied. Unlike standard Koopman operators that learn fixed time-evolution mappings, our approach learns a continuous family of operators K(ν), enabling prediction of plasma evolution for unseen collision parameter values within the learned parameter space [3].

Our cKNO architecture extends the standard formulation by introducing a collision parameter encoder that modulates the spectral evolution operator. The model takes as input: (1) the initial plasma state u_0, (2) the continuous parameter ν, and outputs the evolved state u(t). The key innovation is that the Fourier-domain operator continuously varies with ν, allowing smooth interpolation between learned dynamics while preserving the Koopman framework's linear evolution structure [4].

To formalize the method, we consider a nonlinear dynamical system of the form:

$$\partial\_tu = F(u;,\nu), \qquad (1)$$

where u is the high-dimensional plasma state and ν is the collision parameter. The Koopman operator $\mathcal{K}_\tau(\nu)$ evolves the system linearly in an appropriate observable space such that

$$u(t+\tau) = \mathcal{K}\_\tau(\nu), u(t). \qquad (2)$$

In practice we approximate this evolution in a latent space using an encoder-decoder architecture. The encoder $\mathcal{E}$ maps the input state $u(t) \in R^n$ to a latent representation $z(t) = \mathcal{E}(u(t)) \in R^d$, where $d \ll n$. The decoder $\mathcal{D}$ reconstructs the physical state as $\hat{u}(t) = \mathcal{D}(z(t))$. Time evolution in the latent space is governed by a ν-parameterized Koopman operator $K(\nu) \in C^{d\times d}$, which is treated as a continuous function of the collision parameter. The latent dynamics obey

$$\frac{dz}{dt} = K(\nu)z, \qquad (3)$$

and this ODE is integrated using a differentiable ODE solver such as the midpoint or Runge-Kutta method to yield $z(t)$. This formulation allows end-to-end backpropagation of errors during training through both the encoder-decoder and the time evolution process.

The ν-conditioned continuous Koopman Neural Operator (cKNO) is trained on a discrete set of collisionality parameters and validated on unseen intermediate values. By minimizing reconstruction loss between predicted and simulated states over multiple time steps, the model learns to generalize smoothly across ν. This learned operator manifold not only enables fast, accurate simulations [5] - significantly faster than traditional solvers - but also offers insight into bifurcations and critical thresholds.

References

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