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Energy-consistent discontinuous Galerkin schemes for the visco-resistive magnetohydrodynamic equations

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We are developing MUSES, a three-dimensional, nonlinear magnetohydrodynamic (MHD) code for magnetic confinement fusion that discretizes the MHD equations using the discontinuous Galerkin (DG) method. Our goal is to achieve computational accuracy and numerical stability while triangulating poloidal cross sections. We have performed calculations of the ideal internal kink mode in a torus with a circular cross section and confirmed that the DG method accurately reproduces the linear growth rate and eigenfunctions, indicating its usefulness for magnetically confined plasmas. However, the present analysis is limited to ideal MHD; kinetic and two-fluid effects must be introduced for realistic fusion plasma simulations.

The BR2 method by Bassi et al. has been proposed to discretize the viscosity and thermal conduction in the Navier-Stokes equations [1]. This method adds artificial dissipation to stabilize unphysical oscillations inside cells, and it has been used in various compressible fluid simulations. However, when this method is used to solve resistive MHD equations, it can lead to numerical instabilities, especially in multidimensional simulations. In the case of viscosity and thermal conduction, the dissipative terms are described by divergence forms. In the case of electrical resistance, on the other hand, it is described by rotational operators. If these operators are not discretized carefully, it can lead to the addition of numerical dissipation of the wrong sign. This results in negative entropy production, i.e., conversion from thermal to magnetic energy. Therefore, it is difficult to obtain stable solutions to the resistive MHD equation using existing DG methods, and a new discretization method for electrical resistance must be developed to advance this research.

To address this problem, we try to apply the idea of the KEEP (Kinetic Energy and Entropy Preserving) scheme [2] to electrical resistance. This scheme prevents the generation of negative entropy by determining the numerical flux to satisfy the conservation of kinetic energy in the incompressible limit, and recently we extended the KEEP scheme to the ideal MHD [3]. This guideline of preserving the mathematical properties of the governing equations may be useful not only for advection terms but also for diffusion terms. For example, the conversion from magnetic energy to thermal energy by electrical resistance is proportional to the square of the absolute value of the current, which is always greater than or equal to zero, thereby determining the direction of energy conversion. Preserving these properties even in discrete form is expected to contribute to numerical stability.

We applied the BR2 method to the resistive MHD equations, resulting in the following findings: The physical electrical resistance and the numerical dissipation with respect to it act in the direction of increasing the internal energy. The aliasing error, which corresponds to the oscillation in the cell, has no fixed sign and can be a source of numerical cooling. Therefore, by discretely reproducing the analytical relationship established between the electrical resistance in the induction equation and the Joule heating in the energy equation, the negative entropy production due to aliasing errors can be completely suppressed. In order to compare the proposed algorithm with the BR2 method, these schemes were benchmarked by solving Alfven waves in one-dimensional space. While no significant differences are observed because solving this problem is relatively easy, it was ascertained that the numerical oscillation are mitigated better than the BR2 method for all physical quantities.

Up to this point, the verification has been limited to one-dimensional calculations, but more problems arise in multidimensional calculations. Since electrical resistance is described using rotation operators, unlike the case of divergence operators, it possesses both positive and negative signs. This necessitates a more careful discretization process, with particular attention given to the sign of the numerical dissipation. Furthermore, the solenoidal constraint of the magnetic field can be derived from the divergence of the induction equation, but discretization without attention to this may lead to numerical magnetic monopoles, which may destabilize the simulation. To discuss these issues, we will perform two-dimensional simulations.

References

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