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Relaxation and equilibration of baroclinic flows

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Baroclinic flows are of relevance in rotating stratified systems occurring in geophysical and astrophysical settings. There is an interest in the mechanisms (e.g. inhomogeneous potential vorticity (PV) mixing) and principles (e.g. maximum entropy [1], baroclinic adjustment [2], barotropic governor [3], conservation laws) that control the resulting equilibrated mean profile.

The present study focuses on spin-down experiments in a two-layer quasi-geostrophic system (cf. Hasegawa-Mima equations), initialised with the same baroclinically unstable baroclinic and barotropic PV profile, but where the flow profile itself can change depending on the layer depths, which is the principal system parameter considered in this work. In the limit where the upper layer depth is small, it is found that the flow is driven to a state where the flow is stable by the Charney-Stern condition [4] with no PV gradient reversal, via complete PV homogenisation in the lower layer, consistent with the principle of baroclinic adjustment [2]. The equilibrated profile is predicted well by an optimisation principle detailed in the work of [5], essentially one of potential energy minimisation subject to momentum, energy and PV conservation in the zonal mean profile. The control variables for the problem considered in [5] are the latitudes bands over which PV homogenisation occurs, with constraints enforced by means of Lagrange mutlipliers, and the resulting optimality system can be derived analytically and solved using standard root-finding numerical methods.

Away from the small upper layer limit, however, although there appears to be hints of baroclinic adjustment occurring in the sense that there is still a degree of PV homogenisation, the resulting process appears to be inefficient in the sense that some other process is halting the homogenisation process. The halting behaviour is normally attributed to the barotropic governor [3] (although we note the work of [3] is strictly in the linear regime): as baroclinic instability matures and inhomogeneous PV mixing proceeds, a barotropic jet results that halts the PV mixing, and the flow tends to equilibrate just above marginal criticality (if indeed a linear stability analysis is even valid in the now nonlinear regime).

With an overall aim to understand the interplay between the baroclinic adjustment and barotropic governor principles, we consider in here extensions of the optimisation principle proposed by [5] to take into account of inefficient mixing. We introduce a numerical methodology leveraging automatic code generation software that specifies the optimisation problem at the cost function and constraint level, bypassing the need for an analytical derivation of the optimality system. Principles of varying complexity are explored, and investigation the optimisation principle guides the data analysis to be done in the simulation data and vice-versa.

References

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Figure 1: Pictorial schematic of model set up. The small H_1 limit is dynamically similar to the system considered in [5].

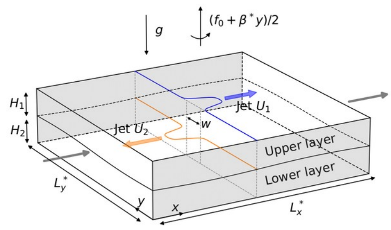

Figure 2: Sample results for the case considered in [5]. Initial zonal mean profiles (black dotted) and optimised zonal mean profiles (solid lines) for (top left) upper layer PV, (top right) bottom layer PV, (bottom left) layer streamfunctions, (bottom right) layer zonal flow.