

Efficient Approaches to Solve Plasma Dispersion Relations with Arbitrary Distributions

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Plasma exhibits a remarkable diversity of waves and instabilities, the linear properties of which are governed by dispersion relations (DR). However, solving these DRs, especially for kinetic waves, poses significant numerical challenges due to singularities and infinite double integrals.

In this work[1,2,3], we present a fast and accurate method for calculating the plasma dispersion function applicable to arbitrary distributions. We then demonstrate how to obtain all solutions of fluid waves and extend this approach to kinetic DRs using a novel matrix transformation method. Starting from the simplest electrostatic 1D case to illustrate the key concepts, we progressively extend the method to more complex scenarios, including the drift bi-Maxwellian electromagnetic case and cases involving ring beams, perpendicular drifts, and collisions. This approach efficiently computes all kinetic solutions while remaining both fast and accurate. A critical breakthrough is achieved by employing well-chosen basis function expansions, which enable the same computational framework developed for Maxwellian distributions to be applied to arbitrary distributions. We validate the method

by demonstrating results for drift ring beam distributions, kappa distributions, and shell distributions, achieving good convergence and agreement with existing benchmarks.

This new approach offers a powerful and efficient tool for solving kinetic DRs with the following key advantages: it is fast, accurate, supports damped modes, provides all solutions, and accommodates arbitrary distributions.

References

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- [2] H.S. Xie, BO: A unified tool for plasma waves and instabilities analysis, Comput. Phys. Comm. 244 (2019) 343-371.
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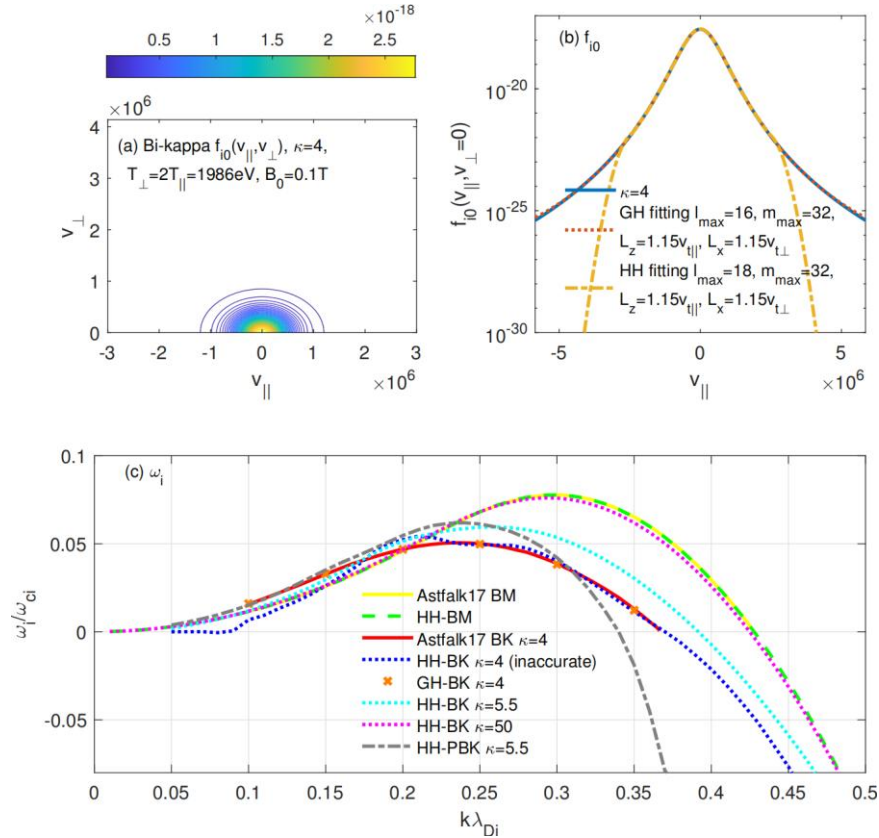


Figure 1. Benchmark of the firehose instability for bi-Maxwellian (BM), bi-kappa (BK), and product bi-kappa (PBK) distributions at $\theta = 45^\circ$, which show that the new solver can support all these distributions well in one model.