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Kinetic equations for strongly magnetized (in)homogeneous plasmas

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The Fokker-Planck collision term to include a uniform magnetic field for homogeneous plasma is derived which has the similar form as the case of no magnetic field as

$$\frac{\partial f_\alpha(\mathbf{v}_\alpha,\tau)}{\partial\tau}+\Omega_\alpha\mathbf{v}_\alpha\times\hat{\mathbf{e}}_z\cdot\frac{\partial f_\alpha(\mathbf{v}_\alpha,\tau)}{\partial\mathbf{v}_\alpha}$$

$$=-\frac{\partial}{\partial\mathbf{v}_\alpha}\cdot[\langle\Delta\mathbf{V}_\alpha\rangle f_\alpha(\mathbf{v}_\alpha,\tau)]+\frac{1}{2}\frac{\partial^2}{\partial\mathbf{v}_\alpha\partial\mathbf{v}_\alpha}:[\langle\Delta\mathbf{V}_\alpha\Delta\mathbf{V}_\alpha\rangle f_\alpha(\mathbf{v}_\alpha,\tau)]$$

but with different Fokker-Planck coefficients $\langle\Delta\mathbf{V}_\alpha\rangle$ and $\langle\Delta\mathbf{V}_\alpha\Delta\mathbf{V}_\alpha\rangle$, which including the magnetic field. The coefficients are calculated explicitly within the binary collision model, which are free from infinite sums of Bessel functions.

The Fokker-Planck approach is employed to derive the kinetic equation for spatially uniform magnetized plasmas. The magnetized Fokker-Planck collision term can be manipulated into the Landau form.

By using the fluctuating electrostatic field for quiescent plasmas, the magnetized Fokker-Planck coefficients are calculated explicitly based on the wave theory which including the collective effects in a proper manner. The magnetized Balescu-Lenard collision term is obtained as

$$\frac{\partial f_\alpha(\mathbf{v}_\alpha,\tau)}{\partial\tau}=\frac{\partial}{\partial\mathbf{v}_\alpha}\cdot\sum_\beta\frac{q_\alpha^2q_\beta^2}{m_\alpha}\int_{-\infty}^{\infty}\mathrm{d}t\int_{-\infty}^{\infty}dt_1\int\mathrm{d}^3\mathbf{k}\int_{-\infty}^{\infty}\mathrm{d}\omega\int\mathrm{d}^3\mathbf{v}_\beta\times$$

$$\exp\{\mathrm{i}\mathbf{k}\cdot[\mathbf{H}_\alpha(t)-\mathbf{H}_\alpha(0)]\cdot\mathbf{v}_\alpha-\mathrm{i}\mathbf{k}\cdot[\mathbf{H}_\beta(t_1)-\mathbf{H}_\beta(0)]\cdot\mathbf{v}_\beta-\mathrm{i}\omega(t-t_1)\}$$

$$\times\frac{\mathbf{T}_\alpha^{-1}(t)\cdot\mathbf{k}}{2(2\pi)^4\varepsilon_0^2|\varepsilon(\mathbf{k},\omega)|^2k^4}\mathbf{k}\cdot\left(\frac{1}{m_\alpha}\frac{\partial}{\partial\mathbf{v}_\alpha}-\frac{1}{m_\beta}\frac{\partial}{\partial\mathbf{v}_\beta}\right)[f_\alpha(\mathbf{v}_\alpha,\tau)f_\beta(\mathbf{v}_\beta,\tau)],$$

where the dielectric response function is

$$\varepsilon(\mathbf{k},\omega)=1+\sum_{\gamma\neq\alpha}\frac{q_\gamma^2}{\varepsilon_0m_\gamma k^2}\int_0^{\infty}\mathrm{d}t\int\mathrm{d}^3\mathbf{v}\,f_\gamma(\mathbf{v},\tau)\times$$

$$\mathbf{k}\cdot[\mathbf{H}_\gamma(t)-\mathbf{H}_\gamma(0)]\cdot\mathbf{k}e^{-\mathrm{i}\mathbf{k}\cdot[\mathbf{H}_\beta(t_1)-\mathbf{H}_\beta(0)]\cdot\mathbf{v}+i\omega t}$$

In which

$$\mathbf{H}_\alpha(t)=\frac{1}{\Omega_\alpha}\begin{pmatrix}\sin(\Omega_\alpha t) & -\cos(\Omega_\alpha t) & 0\\ \cos(\Omega_\alpha t) & \sin(\Omega_\alpha t) & 0\\ 0 & 0 & \Omega_\alpha t\end{pmatrix}.$$

The magnetized Balescu-Lenard equation is identical to the results derived by using the BBGKY hierarchy of equations and the quasilinear method.

The generalized Balescu-Lenard equation is derived from Klimontovich equation for strongly magnetized inhomogeneous plasmas with a collision term incorporating simultaneously the collective interactions, effects of magnetic field and distribution function inhomogeneity, and nonlocality of the collision process by using the quasilinear approach.

The kinetic equation is

$$\frac{\partial f_\alpha}{\partial t}+\mathbf{v}\cdot\frac{\partial f_\alpha}{\partial\mathbf{r}}+\frac{q_\alpha}{m_\alpha}(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\frac{\partial f_\alpha}{\partial\mathbf{v}}=C_\alpha^L+C_\alpha^{NL}$$

Where the local part of collision term is

$$C_\alpha^L=\frac{iq_\alpha^2}{(2\pi)^4\varepsilon_0m_\alpha}\frac{\partial}{\partial\mathbf{v}_\alpha}\cdot\left\{\int\mathrm{d}^3\mathbf{k}\int_{-\infty}^{\infty}\mathrm{d}\omega\int_{-\infty}^{\infty}\mathrm{d}t'\,\frac{\mathbf{k}}{\varepsilon k^2}f_\alpha\times\right.$$

$$\exp\{-\mathrm{i}\mathbf{k}\cdot\mathbf{H}_\alpha^-(t')\cdot\mathbf{v}_\alpha+i\omega t'\}+\sum\frac{q_\beta^2}{\varepsilon_0m_\alpha}\int\mathrm{d}^3\mathbf{k}\int_{-\infty}^{\infty}\mathrm{d}\omega\int_{-\infty}^{\infty}\mathrm{d}t'$$

$$\int_{-\infty}^{\infty}\mathrm{d}t_1\int\mathrm{d}^3\mathbf{v}_1\exp\{-\mathrm{i}\mathbf{k}\cdot[\mathbf{H}_\alpha^-(-t')\cdot\mathbf{v}-\mathbf{H}_\beta(-t')\cdot\mathbf{v}_1]+\mathrm{i}\omega(t_1-t')\}$$

$$\left.\frac{\mathbf{k}\mathbf{k}}{|\varepsilon|^2k^2}\cdot\left[\mathbf{T}_\alpha(-t')\cdot\frac{\partial f_\alpha}{\partial\mathbf{v}}-\mathbf{H}_\alpha^-(-t')\cdot\frac{\partial f_\alpha}{\partial\mathbf{r}}\right]f_\beta\right\}$$

In which

$$\mathbf{T}_\alpha(t)=\frac{1}{\Omega_\alpha}\begin{pmatrix}\cos(\Omega_\alpha t) & \sin(\Omega_\alpha t) & 0\\ -\sin(\Omega_\alpha t) & \cos(\Omega_\alpha t) & 0\\ 0 & 0 & 1\end{pmatrix}$$

Meanwhile the nonlocal part of collision term is

$$C_\alpha^{NL}=-\frac{\partial}{\partial\mathbf{v}}\cdot\frac{q_\alpha^2}{(2\pi)^4\varepsilon_0m_\alpha}\left\{\int\mathrm{d}^3\mathrm{k}\int_{-\infty}^{\infty}\mathrm{d}\omega\int_{-\infty}^{\infty}\mathrm{d}t'\right.$$

$$\mathrm{e}^{-\mathrm{i}\mathrm{k}\cdot\mathrm{H}_\alpha^-(t')\cdot\mathrm{v}+\mathrm{i}\omega t'}\frac{1}{\varepsilon}\frac{\partial\chi_h}{\partial\mathrm{r}}\cdot\frac{\partial}{\partial\mathrm{k}}\frac{\mathrm{k}}{\varepsilon k^2}f_\alpha+\sum_\beta\frac{q_\beta^2}{\varepsilon_0m_\alpha}\mathrm{i}$$

$$\int\mathrm{d}^3\mathrm{k}\int_{-\infty}^{\infty}\mathrm{d}\omega\int_0^{\infty}\mathrm{d}t'\int_{-\infty}^{\infty}\mathrm{d}t_1\int\mathrm{d}^3\mathrm{v}_1\,\mathrm{e}^{\mathrm{i}\omega(t_1-t')}$$

$$\mathrm{e}^{-\mathrm{i}\mathrm{k}\cdot[\mathrm{H}_\alpha^-(-t')\cdot\mathrm{v}-\mathrm{H}_\beta^-(-t_1)\cdot\mathrm{v}_1]}\left\{\frac{\partial f_\beta}{\partial\mathrm{r}}\cdot\frac{\partial}{\partial\mathrm{k}}\left(\frac{\mathrm{k}}{\varepsilon k^2}\right)\right.$$

$$\frac{\mathrm{k}\cdot\mathrm{T}_\alpha(-t')}{\varepsilon^*k^2}\cdot\frac{\partial f_\alpha}{\partial\mathrm{v}}-\frac{\partial\chi_h}{\partial\mathrm{r}}\cdot\frac{\partial}{\partial\mathrm{k}}\left(\frac{\mathrm{k}}{\varepsilon k^2}\right)\frac{\mathrm{k}\cdot\mathrm{T}_\alpha(-t')}{|\varepsilon|^2k^2}$$

$$\cdot\frac{\partial f_\alpha}{\partial\mathrm{v}}f_\beta-\mathrm{i}\frac{\mathrm{k}}{|\varepsilon|^2k^2}\frac{\partial\chi_h^*}{\partial\mathrm{r}}\cdot\left[\mathrm{H}_\alpha^-(-t')\cdot\mathrm{v}+\mathrm{i}\frac{\partial}{\partial\mathrm{k}}\right]$$

$$\left.\left.\frac{\mathrm{k}\cdot\mathrm{T}_\alpha(-t')}{\varepsilon^*k^2}\cdot\frac{\partial f_\alpha}{\partial\mathrm{v}}f_\beta\right\}\right\}.$$

Subsequently, the particle transport process across magnetic field in strongly magnetized plasmas is investigated by using the generalized Balescu-Lenard equation derived above.

References

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