# Dynamic characteristics and formation mechanism of the complex dynamics in subnormal glow discharge systems: Mixed-mode oscillations and period-adding bifurcation

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Laboratory plasmas are intrinsically nonequilibrium dissipative dynamical systems in which the ionization is maintained by the external energy input. Complex spatiotemporal nonlinear dynamic behaviors in laboratory plasmas have long been a central issue of fundamental interest which deepens the understanding of the origin of the instabilities and their mode transitions in the plasma devices. On the other hand, the laboratory plasmas can be treated as a simple and accessible test bed to verify and explore the ubiquitous nonlinear dynamic behaviors such as chaos, self-organization, synchronization, etc. Subnormal glow discharge systems are of great interest because they reveal clearly a very special type of dynamical instability involving homogeneous selfsustained oscillations accompanied by the nonequilibrium fluctuation of plasma parameters.

The negative difference resistance is suggested to be the origin of the intricate temporal instability phenomena. In addition to simple, periodic relaxation oscillations (usually known as self-pulses or self-oscillations) consisting of successive storage and breakdown phases, attention has gradually shifted to the spatiotemporal evolution and formation mechanisms of complex oscillations. The existence of deterministic chaos in laboratory plasmas has been broadly demonstrated. The route to chaos is fascinating; it allows one to understand the transition from regular to nonperiodic oscillations of the system as the bifurcation parameter is varied continuously. Several typical routes to chaos have been observed in laboratory plasmas, such as period-doubling bifurcation, intermittency, and quasiperiodicity. These remarkable results make the laboratory plasmas a realistic physical system for the verification of the universality of the bifurcation and chaos theory.

Mixed-mode oscillations (MMOs), featuring alternating sets of large-amplitude oscillations (LAOs) and small amplitude oscillations (SAOs), are complex dynamic phenomena in multiple timescale slow-fast systems. The subthreshold oscillations and relaxation mechanisms of the system are considered as a basic mechanism responsible for the SAOs and LAOs, respectively. Indeed, the study of MMOs is important because they are a universal phenomenon and have been observed in various scientific fields, including electrochemical system, laser dynamics and plasma physics.

In this work, a period-adding bifurcation sequence of MMOs (i.e., $1^0$-$1^1$-$1^2$-…-$1^n$-$0^1$) was observed in a subnormal glow discharge system by using the ballast resistance as the control parameter, where the number of large-amplitude relaxation oscillations in each oscillation period is fixed and the number of small-amplitude near-harmonic oscillations increases sequentially [1]. Intermittently chaotic regions caused by inverse saddle-node bifurcation exist between adjacent periodic windows, exhibiting an alternating periodicchaotic phenomenon. Time series analyses including the largest Lyapunov exponent and correlation dimension confirmed the deterministic nature of chaotic states and demonstrated that the complexity of strange attractors increases with the order of bifurcations. A self-consistent fluid model reveals that the localized avalanches between the cathode and the virtual anode acted by the bulk plasma play a crucial role in shaping the MMO structure [2]. These localized avalanches arise from the combined effects of rapid voltage increases and delayed bulk plasma dissipation. The findings deepen our understanding of the formation mechanism of the MMOs in subnormal discharge systems and provide helpful inspiration in a wide range of spatially dependent nonlinear dissipative systems.

References

[1] Zijia Chu *et.al*, Physical Review E, 108 (5), 055210 (2023)
[2] Zijia Chu *et.al*, Physical Review E, 111 (5), 055202 (2025)

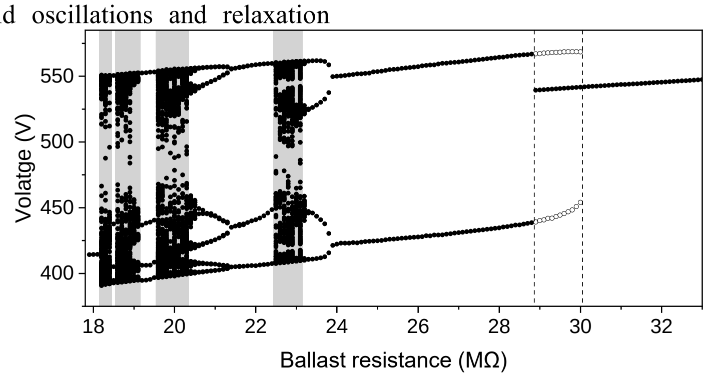


**Figure 1.** The amplitude bifurcaiton diagram of the mixed-mode oscillations with alternating periodic-chaotic sequence