

Complex network in low-temperature plasma analyzed by Shannon entropy

Osamu Sakai¹, Kota Hamano¹¹ Department of Electronic Systems Engineering, The University of Shiga Prefecture

e-mail (speaker): sakai.o@e.usp.ac.jp

1. Introduction

Entropy is at the heart of thermodynamics, and for plasma physics, it has played a significant role in comprehension of magnetohydrodynamical stabilities in magnetized plasma, like space and fusion plasma. Such thermodynamical entropy is equivalent to Shannon entropy, which is simply based on probability distributions without any physical constraints. In this study, we apply Shannon entropy for network statistics and topology of complex chemical reactions in low-temperature plasma.

Networking of several hundreds/thousands of chemical reactions is an emerging promising concept for molecular plasma in industry [1], atmospheric pressure plasma [2,3], and plasma for negative ion sources [4]. In this study, we use entropy for a measure of dispersity of species in a chemical reaction network. Application of entropy for complex networks will open advances in plasma physics as well as other categories in which complexities are main challenges.

2. Method and results

Our theoretical model is based on a classical zero-dimensional rate-equation solution of SiH₄ plasma chemistry [5]. We fixed SiH₄ and electron densities, and 254 chemical reactions are explicitly solved in finite difference formulation. In our previous study [2], the derived web-like reaction network has a scale-free property, which ensures system robustness.

Densities of the total 54 species calculated numerically form a time-varying dataset of species dispersity. When we count number of species on each log-density span, we found that the spectra of the density count fit a log-normal function with the power law effect (a straight line in Fig. 1). Such log-normal distributions are frequently observed in which quantities in a dataset are fractions of one initial unit. That is, in low-temperature plasma, seed species generated by initial decomposition (i.e., SiH_x with $x = 1-3$) can trigger preferential attachment of other species, leading to scale-free network, while, in another aspect, the mother gas is converted into small fractions of species, leading to a log-normal distribution. This statistical measure is in good agreement with entropy, which increases with time (Fig. 1), implying increment of species dispersity.

3. Summary

Complexities in plasma are fundamental since its constituent particles are so abundant that they are in multiple phase spaces. Classical and typical schemes to identify its property are based on statistical analysis, like derivation of energy distribution functions, while recent developments in complex network science provide us useful visualization and analysis tools for general

complex phenomena in which many inner elements are interlinked to each other. Here, a key parameter we can estimate is entropy, which has played significant roles in space and fusion plasma science for understanding magnetohydrodynamical stability, is also a key statistical measure of such complex chemical networks not as thermodynamic entropy but as Shannon entropy.

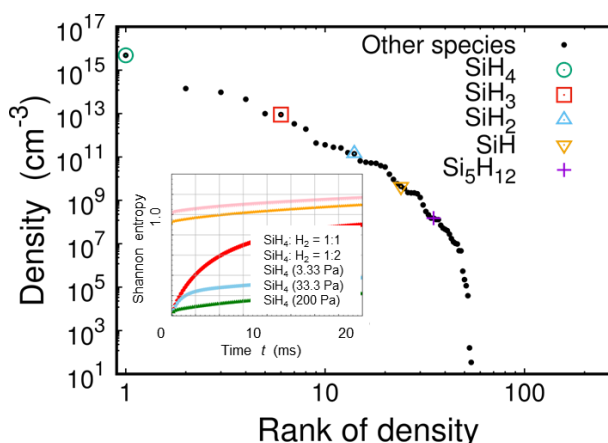


Figure 1. Rank-size plot of species densities in SiH₄ plasma after 10-ms discharge. The inset is Shannon entropy evolution in discharge time in various gases [5].

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References

- [1] O. Sakai, K. Nobuto, S. Miyagi and K. Tachibana: Analysis of weblike network structures of directed graphs for chemical reactions in methane plasmas. *AIP Adv.* **5**, 107140 (2015).
- [2] T. Murakami and O. Sakai: Rescaling the complex network of low temperature plasma chemistry through graph theoretical analysis. *Plasma Sources Sci. Technol.* **29**, 115018 (2020).
- [3] T. D. Holmes, R. H. Rothman and W. B. Zimmerman: Graph theory applied to plasma chemical reaction engineering. *Plasma Chem. Plasma Process.* **41**, 531 (2021).
- [4] S. Venturi, W. Yang, I. Kaganovich and T. Casey: An uncertainty-aware strategy for plasma mechanism reduction with directed weighted graphs. *Phys. Plasmas* **30**, 043904 (2023).
- [5] K. Hamano, S. Miyagi, T. Murakami, T. Ito and O. Sakai: Statistical measure of dispersity in complex weighted networks of low temperature plasma chemistry *J. Phys. Complex.* **6**, 015019 (2025)