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Application and progress of Bayesian statistics in plasma physics

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We introduce the mathematics of parametric and nonparametric regression based on Bayesian statistics, as well as its application to plasma diagnostics. We also briefly review our collaborative study, which includes a model selection method for ion velocity distribution functions [1], a statistical theory for noisy data [2], an inference method for plasma parameter profiles and their derivatives [3], and its expansions [4,5].

Physicists often describe a phenomenon of interest by using a model that separately represents universal and individual aspects of the phenomenon through a function (or equation), along with its parameters. Parametric regression is applied to estimate a finite set of parameters in such models by comparing them to experimental data. Conversely, nonparametric regression does not assume explicit (finite-parametric) forms for modeling data. It is applied to interpolate experimental data that is governed by unknown physics or hard-to-solve equations.

As an example of parametric regression, we show Bayesian inference of the ion velocity distribution function [1]. The Maxwellian distribution is the most fundamental velocity distribution function, which holds for systems in thermodynamic equilibrium states. However, the ions in magnetized plasmas are usually far from equilibrium. Intricate factors in non-equilibrium states, such as anisotropy, energy transfer, and symmetry-breaking deform their distribution. While experimentally establishing the explicit expression of distribution is crucial for understanding the system of interest, it is often ambiguous which factor is dominant. We propose a new approach to resolve such an ambiguity that combines laser spectroscopy and Bayesian inference. We have demonstrated our approach by applying it to laser-induced fluorescence spectra observed locally at several positions in a linear magnetized plasma.

We also show a mathematical perspective on parametric regression with noisy experimental data. Estimating parameters and selecting models depend on the quantity and quality of data. We have developed an asymptotic theory of Bayesian inference that considers the effects of observation noise by advancing theoretical analysis based on the mathematical correspondence between Bayesian statistics and the statistical mechanics of disordered systems [2]. We discovered that Bayesian inference involves multiple “phases,” depending on the quality and quantity of data; the higher the quantity and quality of the data, the more complex the model with more parameters is deemed to be the best.

As an example of non-parametric regression, we demonstrate the estimation of plasma parameter profiles and their derivatives using Gaussian processes. We have developed a new Gaussian process regression (GPR) technique that provides the probability distributions of a parameter profile and its derivative based on any linear observations [3]. By applying this method, we have successfully evaluated the electron density profile and its derivative in a fusion plasma from line-integrated measurements. In the conventional GPR, such an inference is not possible since the finite spatial resolution of diagnostics is neglected, and the observables are assumed to represent the parameter of interest with no blur or integrating effect. The new GPR expands the capabilities, allowing measurements with poor spatial resolution or even line-integrated ones to be utilized.

We also present variants of our GPR approach, including equilibrium reconstruction of axisymmetric plasmas [4] and inference of flow shear from reciprocating plasma potential measurements [5]. They provide a fresh perspective on methodological studies. The former sheds light on the inference of parameter profiles by integrating our GPR approach and the governing equations as “virtual observations”. The latter contributes to incorporating the adequate treatment of the plasma fluctuations into the inference of parameter profiles based on the GPR approach.

References

[1] S. Tokuda, Y. Kawachi, M. Sasaki, H. Arakawa, K. Yamasaki, K. Terasaka, and S. Inagaki, “Bayesian inference of ion velocity distribution function from laser-induced fluorescence spectra”, *Sci. Rep.* **11**, 20810 (2021).

[2] Satoru Tokuda, Kenji Nagata, and Masato Okada, “Intrinsic regularization effect in Bayesian nonlinear regression scaled by observed data”, *Phys. Rev. Res.* **4**, 043165 (2022).

[3] T. Nishizawa, S. Tokuda, T. Kobayashi, K. Tanaka, H. Funaba, I. Yamada, Y. Takemura, T. Tokuzawa, R. Yasuhara, H. Uehara, K. Ida, H. Takahashi, Y. Kawachi, S. Inagaki, S. Kado, M. Sasaki, and A. Fujisawa, “Estimation of plasma parameter profiles and their derivatives from linear observations by using Gaussian processes”, *Plasma Phys. Control. Fusion* **65**, 125006 (2023).

[4] T. Nishizawa, S. Tokuda, A. Sanpei, M. Hasegawa, K. Yamasaki, and A. Fujisawa,"Equilibrium reconstruction of axisymmetric plasmas by combining Gaussian process regression and Markov chain Monte Carlo sampling", *Plasma Phys. Control. Fusion* **67**, 015006 (2024).

[5] T. Nishizawa, P. Manz, S. Tokuda, G. Grenfell, M. Sasaki, S. Inagaki, Y. Kawachi, and A.Fujisawa, “Inference of flow shear from reciprocating plasma potential measurements by means of Gaussian process regression”, *Phys. Plasmas* **32**, 032507 (2025).