

Phase transition from hydrodynamic turbulence to zonal flows and back

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The modified Hasegawa-Wakatani system can be considered as a minimal model of drift wave turbulence with waves, flows and linear instability. Since it conserves potential vorticity, it has the structure of a beta-plane model, with energy injected internally by the instability mechanism instead of by forcing. It forms zonal flows, and provides a transition between two dimensional hydro-dynamic like turbulence, and a quasi-one-dimensional zonostrophic turbulence. Defining the ratio of the adiabaticity parameter to the background density gradient as the control parameter and the zonal to total kinetic energy as the order parameter for the flow, it is observed that this transition displays characteristic features of phase transitions, and exhibits a hysteresis loop around the transition point[1].

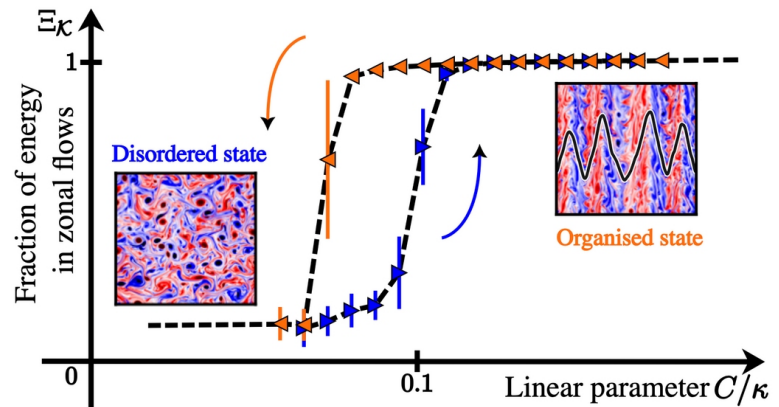
One interesting aspect of this transition is that at the limit of high adiabaticity parameter, the system provides a third conserved quantity, sometimes called the zonostrophy [2], which is conserved by the wave dynamics. The existence of this third conservation is sometimes invoked to explain the anisotropic “inverse” cascade of energy towards zonal flows.

In this sense the transition from isotropic two dimensional turbulence to quasi-one-dimensional turbulence, is a transition from double cascade to triple cascade (with three independent conservation laws), in the same spirit that the transition from three dimensional turbulence to two dimensional one, is a transition from forward cascade to dual cascade.

Further results including a simple quasi-linear saturation rule which accounts for the presence of zonal flows, a discussion of local linear analysis and its implications especially for the zonal flow dominated case will be discussed.

Estimations for the wave-number spectrum in various cases, as well as a detailed study of linear analysis in the presence zonal flows are performed in order to characterize the turbulence in this system. It is concluded that, for a moderate value of the control parameter, the large scales can be dominated by zonal flows, while small scales basically present a 2D turbulence, with forward enstrophy cascade in the presence of those large scale zonal flow patterns.

Finally, considerations about fixed gradient vs. flux-driven formulations, and some preliminary results from a flux-driven code, using the penalisation technique are presented.



References:

- [1] P. L. Guillon and Ö. D. Gürcan *Phys. Plasmas* **32**(1) 012306 (2025)
- [2] A. M. Balk *et al.* *Physics Letters A*, **152**, 276 (1991)
- [3] Ö. D. Gürcan, arxiv <https://doi.org/10.48550/arXiv.2403.09911>
- [4] https://github.com/gurcani/hwak_cuda
- [5] <https://github.com/piergui/P-FLARE>