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Interplay between nonlinear transport crossphase and zonal modes in two-field ITG turbulence

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It is well-known that $E \times B$ flows can suppress turbulent transport in fusion devices, either via suppression of the turbulence amplitude [1], or via suppression of the *transport crossphase* between the electric potential and the transported quantity, e..g. ion/electron temperature or density, etc ... [2].
Several works showed evidence of radial modulation of the transport crossphase in the Hasegawa-Wakatani model [3,4,5]. In the present work, we derive the nonlinear crossphase dynamics directly from the Tensor wave-kinetic equation (TWKE) [6] - a natural extension of the standard WKE for drift-waves [7]. We apply this analysis to the well-known Chalmers two-field ion-temperature-gradient driven (ITG) turbulence model [8,9]. The trace of this equation recovers the standard scalar WKE in the drift-wave limit, while the off-diagonal terms involve the transport crossphase ζ_k between ion temperature T_k and radial velocity $v_{rk} = -ik_\theta \phi_k$ fluctuations (related to electric potential ϕ_k) and the associated amplitude ratio $\beta_k = \frac{|T_k|}{|\phi_k|}$. Physically, the associated Wigner tensor W is related to the plasma entropy S via $S = \frac{1}{2} \ln \det W$, with 'det' the determinant of the tensor [10]. For the two-field Chalmers ITG model, the Wigner tensor can be written in the form:

$$W = \begin{bmatrix} N_k & ir_k e^{-i\zeta_k} \\ -ir_k e^{i\zeta_k} & \beta_k^2 N_k \end{bmatrix}, \quad (1)$$

with $r_k = \beta_k N_k$, and $N_k = [1 + k_\perp^2]^2 |\phi_k|^2$ is the wave action density for ITG turbulence, i.e. potential enstrophy density. The off-diagonal terms in the Wigner tensor are responsible for driving turbulent heat transport. The ITG *ion heat flux* is expressed in the form: $Q_i = \sum \frac{k_\theta}{1+k_\perp^2} \langle \beta_k N_k \cos \zeta_k \rangle$
Zonal flows V_{ZF} and zonal ion temperature T_{zon} are described via the perturbed Hamiltonian, i.e. nonlinear advection frequency. The latter has diagonal terms $\sim k_\theta V_{ZF}$ and off-diagonal terms $\sim T'_{zon}$, where the prime indicates radial derivative.
After some algebra, one can show that the radial phase-shift between zonal flow shear and zonal temperature, i.e. the *zonal crossphase* ζ_q *evolves as:*
$\frac{d\zeta_q}{dt} = \Omega_q^{res} - Re(\Omega_q) - \frac{q_r}{\beta_q} Im[\frac{\delta Q}{\delta V'_q} e^{i\zeta_q}]$, with q_r the radial wavenumber of zonal flows, V'_q the ZF shear, and β_q the zonal amplitude ratio. Here, Ω_q is the complex-valued zonal frequency, $\Omega_q^{res} = q_r c_q$ denotes the zonal resonance frequency, with $c_q = Re[\frac{\delta Q}{\delta T_q}]$ the radial propagation speed. The zonal crossphase dynamics is similar to the Kuramoto equation [11]. Hence, phase-locking occurs and drives zonal temperature corrugations, provided the turbulence phase coherence is large enough. The ITG crossphase also evolves via a Kuramoto-like equation, which we call the *phase kinetic equation.* It is affected both by zonal flow shear and zonal temperature curvature, via shearing effects, but also by zonal temperature gradient which induces local flattening/steepening of the temperature profile and hence modulation of the ITG drive. Without zonal flows, the ITG crossphase dynamics reduces to:
$\frac{\partial \zeta_k}{\partial t} = -\Delta\omega N_k [\tan \zeta_k - \tan \zeta_{k0}]$,
with ζ_{k0} the phase-locked solution. For ITG the relaxation rate is proportional to the turbulent decorrelation rate $\Delta\omega$, as opposed to the collision frequency for dissipative modes. When taking into account zonal modes, they suppress the crossphase, via k-space diffusion.

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