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Reconsideration of unified electron temperature scale via power-law scaling using nonextensive statistical mechanics

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1. Introduction

In non-equilibrium plasmas, their statistical properties were observed to deviate from the standard Maxwell–Boltzmann distribution. As a result, determining the electron temperature in non-equilibrium plasmas proves challenging unless the electron energy distribution function (EEDF) is approximated as an ideal Maxwell-Boltzmann distribution [1,2], where the slope of the Boltzmann plot is directly related to the temperature based on traditional Boltzmann-Gibbs statistics [3]. To overcome this problem, nonextensive statistical mechanics, which extends exponential distributions to power-law distributions [4], was applied to investigate the temperature. This study may open new pathway to understand plasma properties using an additional parameter q expanding the meaning of temperature T.

2. Theory and Results

To calculate the EEDF in low-temperature weakly ionized He plasmas, the Boltzmann equation, which accurately incorporates the effects of excitation and ionization due to collisions, was used. The EEDF for the He plasma was computed using BOLSIG+ [5] as shown in the Boltzmann plot Fig. 1.

Tsallis entropy serves as one-parameter extensions that reflect the degree of departure from Boltzmann-Gibbs statistics.

$$S_q = k\,\frac{1-\sum_i^k p_i^q}{q-1} = -k\sum_{i=1}^{N} p_i^q \ln_q p_i \qquad (1)$$

This entropy agrees with the traditional Gibbs entropy in the limit $q \rightarrow 1$. Consequently, this method enables the description of its entropy, which fundamentally follows a power-law distribution instead of a traditional exponential distribution.

In nonextensive statistical mechanics, the Tsallis distribution can be derived by maximizing the Tsallis entropy, which is a non-additive extension of the Gibbs entropy, under the constraint of q-averaged energy. Using this probability distribution, the temperature in Tsallis statistical mechanics is defined as follows [6].

$$T_{(Tsallis)} = \left[1+(1-q)S_q\right]\cdot\frac{\partial U_q}{\partial S_q} = \frac{1}{k\beta_{ave}} \qquad (2)$$

EEDF was fitted using the power-law-shaped probability distributions of the Tsallis distribution. The temperature calculated based on non-extensive statistical mechanics is illustrated in Fig. 2. For the plasma parameters investigated in this study, the two temperatures in Eq. (1) derived from the Tsallis statistical mechanics coexisted.

3. Conclusion

This confirms that consistent temperatures can be obtained within the framework of non-extensive statistical mechanics.

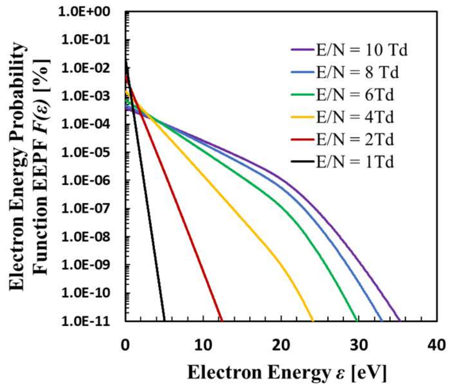

Figure 1 electron energy probabilistic function (EEPF) of the weakly ionized He plasma.

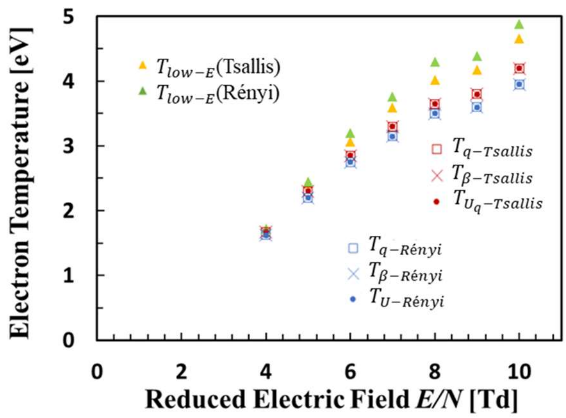

Figure 2 Comparison between electron temperatures based on non-extensive Tsallis statistics plotted against reduced electric field.

4. References

[1] R. Alvarez, J. Cotrino, and A. Palmero, EPL **105**, 15001 (2014).
[2] H. Akatsuka and Y. Tanaka, Entropy. **25**, 276 (2023).
[3] C. Tsallis, Introduction to Nonextensive Statistical Mechanics (Springer, New York, 2009).
[4] K. Kikuchi and H. Akatsuka, Entropy. **25**, 1400 (2023).
[5] G. J. M. Hagelaar and L. C. Pitchford, Plasma Sources Sci. Technol. **14**, 722 (2005).
[6] C. Tsallis, J. Stat. Phys. **52**, 479 (1988).