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Hasegawa-Mima equations and Rossby waves in Geophysical Fluid Dynamics

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Hasegawa & Mima (1978) introduced a 2D equation which describes the time development of the electric potential ϕ of drift waves in a plasma under the strong vertical magnetic field slowly varying in space.

$$\frac{\partial}{\partial t}(\nabla^2\phi - \phi) - [(\nabla\phi \times \boldsymbol{e_z}) \cdot \nabla][\nabla^2\phi - \ln n_0] = 0,$$

where n_0 is the plasma density.

In geophysical fluid dynamics (GFD), on the other hand, Cherney (1948) had derived a quasi-geostrophic equation for geophysical fluids (atmosphere/ocean) under the beta-plane approximation,

$$\frac{\partial}{\partial t}\left(\nabla^2\psi - \frac{1}{L_D^2}\psi\right) - [(\nabla\psi \times \boldsymbol{e_z}) \cdot \nabla][\nabla^2\psi + \beta y] = 0,$$

which describes the beta-plane flow field, where ψ is the stream function, β the latitudinal variation rate of Coriolis parameter, L_D the Rossby deformation radius (for barotropic case, $L_D = \infty$) and y-axis is taken northward. Hasegawa-Mima and Charney equations are quite similar to each other in their mathematical forms, and are sometimes both called Charny-Hasegawa-Mima equation in spite of their physical differences.

In GFD context, Charney equation has attracted interest because its turbulence shows clear anisotropy as east-west zonal flows emerging even when the initial field is random and isotropic (Rhines 1970). Each Fourier mode corresponds a linear solution (Rossby wave) of the linearized equation. The Rossby wave has its own time scale (ω_R^{-1}) due to its anisotropic dispersion relation, while the turbulent field has another time scale (ω_T^{-1}) determined by isotropic energy inverse cascade.

In large wavenumber region ($\omega_T^{-1} \ll \omega_R^{-1}$), the turbulent flow is isotropic and has properties similar to normal two-dimensional turbulence, but in small wavenumber region ($\omega_T^{-1} \sim \omega_R^{-1}$) the flow is quite anisotropic and the 2D energy spectrum shows a clear dumbbell region of little energy (Fig.1(a)). Numerical simulations show that as time goes on, the energy slides into the gap between two “weights” of the dumbbell, which brings about the east-west zonal flow formation. The dumbbell shape has been discussed by balancing the time scales of turbulence and Rossby waves (Vallis & Maltrud 1993). We show the dumbbell shape is obtained with no adjustable parameters (MY and Obuse 2025) by taking into account the Rossby waves in resonance with the initially excited wavenumbers (Fig.1 (b,c)).

Once the zonal flow emerges, weakly nonlinear theory describes the interaction of the Rossby waves with the zonal flow, accelerating zonal flow by dropping the westward momentum at the critical layer under the effect of (eddy) viscosity (Hayashi et al. 1999). In fully nonlinear stage, the potential vorticity distribution plays an important role in time-development of the zonal flow.

On a rotating sphere, the Charney equation ($L_D = \infty$) takes the form of

$$\frac{\partial}{\partial t}\nabla^2\psi - [(\nabla\psi \times \boldsymbol{e_r})] \cdot \nabla[\nabla^2\psi + 2\Omega\mu] = 0,$$

where μ is sine of latitude, Ω the rotation rate, and then the Rossby wave is described by the spherical harmonics. Numerical study shows that strong westward zonal flows are only formed in the circumpolar regions and no strong zonal flows survive in low- and mid-latitudes, while under some forcing mechanism, they survive but gradually merges with each other leaving only a few large jets in low- and mid-latitudes (Obuse et al. 2010). These results do not agree with the observation on the planets as Jovian and Saturn, which suggests the deep convection is important for the zonal jets in giant gas planets, also as suggested in recent precise gravity observation by satellites (Kaspi et al. 2023).

References

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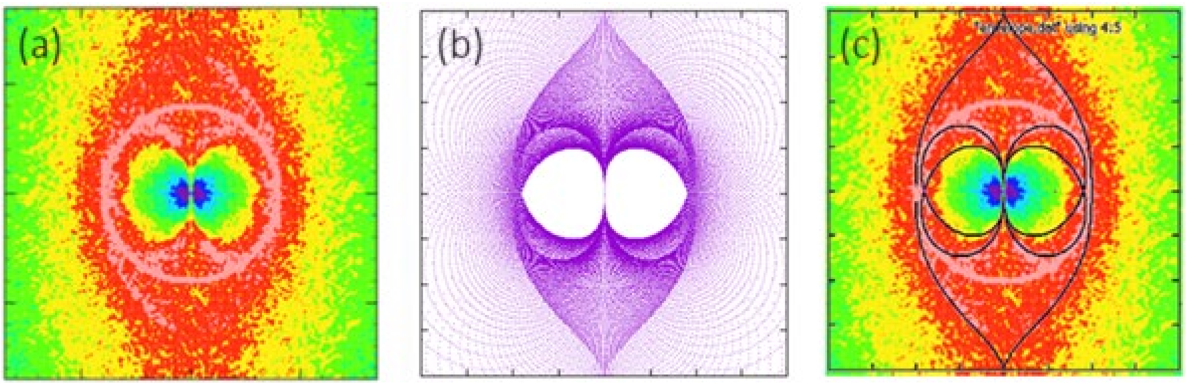

Figure 1: (a) Dumbbell shape of 2D energy spectrum of beta-plane turbulence ($L_D = \infty$) with the initial energy on a unit circle, (b) Distribution of resonant wavenumbers, (c) Envelopes (black curves) of the resonant-wavenumber distribution.